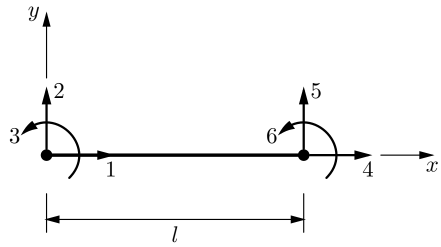
Stroga metoda deformacija

$$\begin{bmatrix} \frac{EA}{l} & 0 & 0 & -\frac{EA}{l} & 0 & 0 \\ 0 & \frac{12EI}{(1+\Phi)l^3} & \frac{6EI}{(1+\Phi)l^2} & 0 & -\frac{12EI}{(1+\Phi)l^3} & \frac{6EI}{l^2} \\ 0 & \frac{6EI}{(1+\Phi)l^2} & \frac{4EI}{(1+\Phi)l} & 0 & -\frac{6EI}{(1+\Phi)l^2} & \frac{2EI}{l} \\ -\frac{EA}{l} & 0 & 0 & \frac{EA}{l} & 0 & 0 \\ 0 & -\frac{12EI}{(1+\Phi)l^3} & -\frac{6EI}{(1+\Phi)l^2} & 0 & \frac{12EI}{(1+\Phi)l^3} & -\frac{6EI}{l^2} \\ 0 & \frac{6EI}{(1+\Phi)l^2} & \frac{2EI}{(1+\Phi)l} & 0 & -\frac{6EI}{(1+\Phi)l^2} & \frac{4EI}{l} \end{bmatrix}$$



Proizvoljan položaj štapova
Transformacija iz lokalnog u globalni koordinatni sistem $\boldsymbol{K}_{glob} = \boldsymbol{T}^T \boldsymbol{K}_{loc} \boldsymbol{T}$

Stroga metoda deformacija sa zanemarenjem deformacije smicanja

Gornja matrica krutosti sa $\Phi = 0$

Tehnička metoda deformacija
Štap aksijalno krut ⇔ zanemarena podužna deformacija

$$\frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix}$$

$$\frac{EI}{l^3}\begin{array}{c} \begin{array}{cccc} k_1 & & k_2 & k_3 \end{array} \\ \begin{bmatrix} 3 & 0 & -3 & 3l \\ 0 & 0 & 0 & 0 \\ -3 & 0 & 3 & -3l \\ 3l & 0 & -3l & 3l^2 \end{bmatrix} \end{array}$$

Pošto su štapovi horizontalni ili vertikalni, izbjeći ćemo premnožavanje sa matricom transformacije

Štapovi horizontalni ili vertikalni

$$\frac{EI}{l^3}\begin{array}{c} \begin{array}{cccc} k_1 & k_2 & k_3 & \end{array} \\ \begin{bmatrix} 3 & 3l & -3 & 0 \\ 3l & 3l^2 & -3l & 0 \\ -3 & -3l & 3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

Moguće je formirati i matrice krutosti za štap sa otpuštenom poprečnom silom, što će biti dato u zadacima

Metoda zaokreta uglova

$$\frac{EI}{l}\begin{bmatrix} 4 & 2 \\ 2 & 4 \end{bmatrix}$$

Matrica transformacije $\boldsymbol{T} = \boldsymbol{I}$
Rotacija ostaje nepromijenjena pri rotaciji koordinatnog sistema u ravni pa ovu matricu krutosti nije potrebno premnožavati sa matricom transformacije

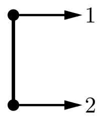
Proizvoljan položaj štapova

$$\begin{bmatrix} \frac{3EI}{l} \end{bmatrix}$$

Metoda krutih rigli, a vertikalnih elastičnih stubova (ad hoc naziv; ovakav okvir se zove i "smičuća greda")

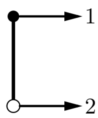
$$\frac{12EI}{l^3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

Biće korišteno kod dinamičkog proračuna gdje ćemo (da bismo smanjili broj stepeni slobode kretanja) pretpostaviti da su grede potpuno krute a stubovi aksijalno kruti.



Štap vertikalan kod okvira sa krutom gredom

$$\frac{3EI}{l^3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



Rešetka

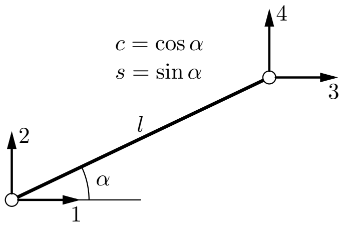
$$\frac{EA}{l}\begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix}$$

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \begin{bmatrix} u_{n1} \\ u_{n2} \end{bmatrix}$$

$c = \cos\alpha$
$s = \sin\alpha$

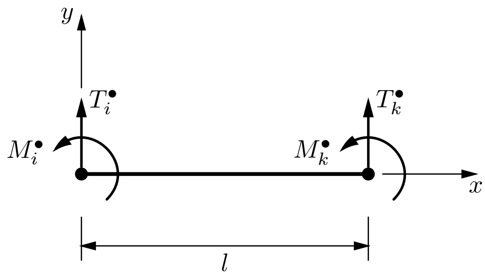
$$= \begin{bmatrix} c & 0 \\ s & 0 \\ 0 & c \\ 0 & s \end{bmatrix} \frac{EA}{l} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} c & s & 0 & 0 \\ 0 & 0 & c & s \end{bmatrix}$$

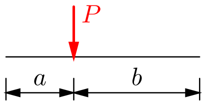
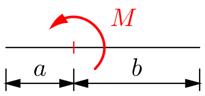
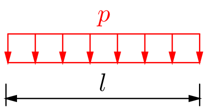
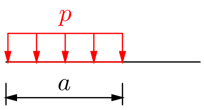
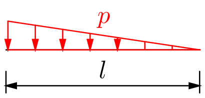
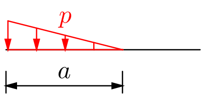
Matrica krutosti već premnožena sa matricom transformacije



Brojevima su označeni lokalni stepeni slobode kretanja. (Važi za sve.)

Slika 5.1: Matrice krutosti u *lokalnom* koordinatnom sistemu (osim štapa rešetke) za najčešće slučajeve idealizacija ravnih linijskih konstrukcija. Matrica krutosti elementa u *globalnom* koordinatnom sistemu se dobija transformacijom komponentalnih krutosti iz lokalnog u globalni koordinatni sistem dakle premnožavajući matricu krutosti sa matricom transformacije: $\boldsymbol{K}_e^{glob} = \boldsymbol{T}^T \boldsymbol{K}_e^{lok} \boldsymbol{T}$. U slučaju kad se radi o horizontalnim i vertikalnim štapovima elementi matrice $\boldsymbol{T}$ su 1 i −1 što nam omogućava da efikasno (a bez računara) formiramo globalnu matricu konstrukcije. U slučaju rešetke zbog proizvoljnosti položaja štapova ne možemo izbjeći matricu transformacije $\boldsymbol{T}$ ali možemo unaprijed proračunati $\boldsymbol{T}^T \boldsymbol{K}_e^{lok} \boldsymbol{T}$ što je gore i dato.

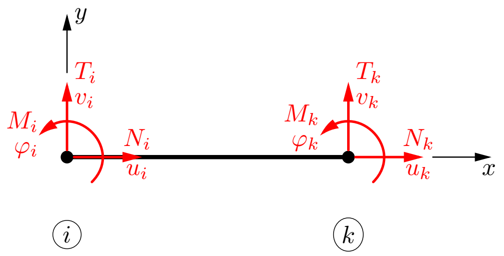


	$M_i^\bullet$	$M_k^\bullet$	$T_i^\bullet$	$T_k^\bullet$
 P, a, b	$-P\dfrac{ab^2}{l^2}$	$P\dfrac{a^2b}{l^2}$	$-\dfrac{Pb^2}{l^3}(3a+b)$	$-\dfrac{Pa^2}{l^3}(a+3b)$
 M, a, b	$-M\dfrac{b}{l^2}(2a-b)$	$-M\dfrac{a}{l^2}(2b-a)$	$-6M\dfrac{ab}{l^3}$	$6M\dfrac{ab}{l^3}$
 p, l	$-\dfrac{pl^2}{12}$	$\dfrac{pl^2}{12}$	$-\dfrac{pl}{2}$	$-\dfrac{pl}{2}$
 p, a	$-\dfrac{pa^2}{12l^2}\left[2l(3l-4a)+3a^2\right]$	$\dfrac{pa^3}{12l^2}(4l-3a)$	$\dfrac{pa^2}{2l^3}\left(2al-a^2-\dfrac{2l^3}{a}\right)$	$\dfrac{pa^2}{2l^3}\left[(l-a)^2-l^3\right]$
 p, l	$-\dfrac{pl^2}{20}$	$\dfrac{pl^2}{30}$	$-\dfrac{7pl}{20}$	$-\dfrac{3pl}{20}$
 p, a	$-\dfrac{pa^2}{60l^2}(10bl+3a^2)$	$\dfrac{pa^3}{60l^2}(5b+2a)$	$\dfrac{pa}{60l^3}(5a^2b-20al^2-a^3$ $-10abl-30bl^2)$	$\dfrac{pa^3}{12l^3}(a-b-2l)$
T_g / T_d $\Delta T=T_d-T_g$ $T_0=(T_d+T_g)/2$	$-EI\alpha_t\dfrac{\Delta T}{h}$	$EI\alpha_t\dfrac{\Delta T}{h}$		

Slika 5.2: Evivalentno opterećenje na štapu kruto vezanom na oba kraja

5.1.1 Matematska konvencija o predznaku presječnih sila

U metodi deformacija koristimo matematsku konvenciju o predznacima presječnih sila. Pozitivne sile i pomjeranja su dati na slici 5.4



Slika 5.4: Matematska konvencija o predznacima pomjeranja i sila - pozitivni smjerovi.

Dakle ako kao rezultat dobijemo vektor sila

$$\begin{bmatrix} T_i \\ M_i \\ T_k \\ M_k \end{bmatrix} = \begin{bmatrix} 0.00 \\ 54.34 \\ 0.00 \\ -54.34 \end{bmatrix}$$

dobili smo konstantan momenat duž štapa.

5.2 Proračun presječnih sila na krajevima štapova

Presječne sile na krajevima štapa proračunavamo sa

$$\boldsymbol{f}^e = \boldsymbol{K}^e \boldsymbol{u}^e - \boldsymbol{s}_e^{\bullet} \tag{5.1}$$

gdje je

- $\boldsymbol{f}^e$ vektor sila na krajevima štapa,
- $\boldsymbol{K}^e$ matrica krutosti elementa,
- $\boldsymbol{u}^e$ vektor pomjeranja čvorova štapa,
- $\boldsymbol{s}_e^{\bullet}$ vektor ekvivalentng opterećenja na kruto vezanom štapu dat u tabeli 5.2 za neke najčešće slučajeve opterećenja.

5.2.1 Štap kruto vezan na oba kraja

$$M_{ik} = k_{ik}\left(4\varphi_i + 2\varphi_k - 6\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\bullet}$$
$$M_{ki} = k_{ik}\left(2\varphi_i + 4\varphi_k - 6\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ki}^{\bullet}$$
$$T_{ik} = \bar{k}_{ik}\left(6\varphi_i + 6\varphi_k - 12\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\bullet}$$
$$T_{ki} = \bar{k}_{ik}\left(-6\varphi_i - 6\varphi_k + 12\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\bullet}$$

gdje je $k_{ik} = \dfrac{EI}{l_{ik}}$ i $\bar{k}_{ik} = \dfrac{EI}{l_{ik}^2} = \dfrac{k_{ik}}{l_{ik}}$

5.2.2 Štap kruto vezan u čvoru i a zglobno u čvoru k

$$\boldsymbol{f}^e = \boldsymbol{K}^e \boldsymbol{u}^e - \boldsymbol{s}_e^{\circ} \tag{5.2}$$

$$M_{ik} = k_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$
$$M_{ki} = 0$$
$$T_{ik} = \bar{k}_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$
$$T_{ki} = -\bar{k}_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$
$$\varphi_k = -0.5\varphi_i + 1.5\frac{\Delta_{ik}}{l_{ik}} + 0.25\frac{M_{ki}^{\bullet}}{k_{ik}} \tag{5.3}$$

Vektor $\boldsymbol{s}_e^{\bullet}$ predstavlja vektor ekvivalentnog opterećenja na štapu kruto vezanom na oba kraja, dok sa oznakom $\boldsymbol{s}_e^{\circ}$ označavamo evivalentno opterećenje na štapu sa otpuštenom nekom vezom, u gornjem slučaju je to zglobna veza u čvoru k.

$$\boldsymbol{s}_e^{\bullet} = \begin{bmatrix} T_{ik}^{\bullet} \\ M_{ik}^{\bullet} \\ T_{ki}^{\bullet} \\ M_{ki}^{\bullet} \end{bmatrix} \qquad \boldsymbol{s}_e^{\circ} = \begin{bmatrix} T_{ik}^{\circ} \\ M_{ik}^{\circ} \\ T_{ki}^{\circ} \\ M_{ki}^{\circ} \end{bmatrix}$$

Veza između sila na zglobno i kruto vezanom štapu postoji pa je za gornji slučaj

$$M_{ik}^{\circ} = M_{ik}^{\bullet} - \frac{1}{2}M_{ki}^{\bullet}$$
$$M_{ki}^{\circ} = 0$$
$$T_{ik}^{\circ} = T_{ik}^{\bullet} - 1.5\frac{M_{ki}^{\bullet}}{l_{ik}}$$
$$T_{ki}^{\circ} = T_{ki}^{\bullet} + 1.5\frac{M_{ki}^{\bullet}}{l_{ik}}$$

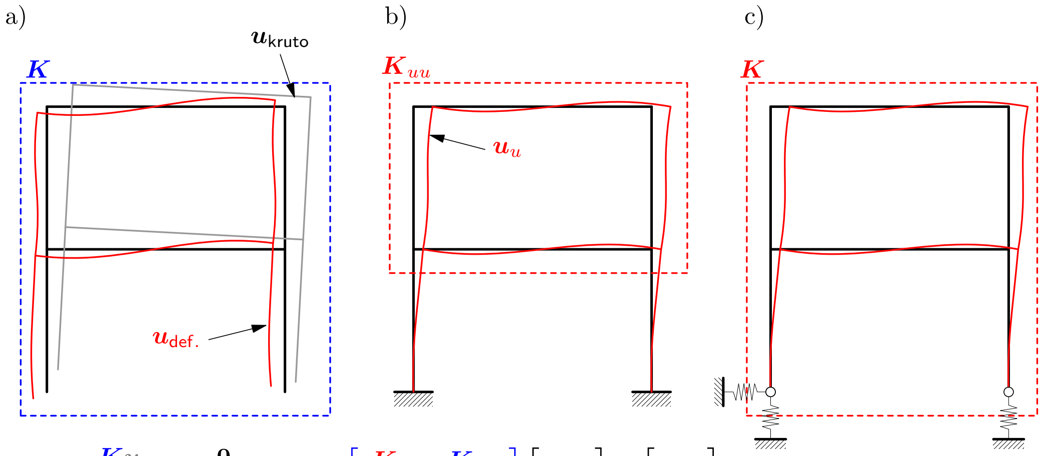
5.2.3 Štap kruto vezan u čvoru k a zglobno u čvoru i

$$M_{ik} = 0$$
$$M_{ki} = k_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$
$$T_{ik} = \bar{k}_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$
$$T_{ki} = -\bar{k}_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

$$\varphi_i = -0.5\varphi_k + 1.5\frac{\Delta_{ik}}{l_{ik}} + 0.25\frac{M_{ik}^{\bullet}}{k_{ik}}$$

Veza između sila na zglobno i kruto vezanom štapu postoji pa je za gornji slučaj

$$M_{ik}^{\circ} = 0$$
$$M_{ki}^{\circ} = M_{ki}^{\bullet} - \frac{1}{2}M_{ik}^{\bullet}$$
$$T_{ik}^{\circ} = T_{ik}^{\bullet} - 1.5\frac{M_{ik}^{\bullet}}{l_{ik}}$$
$$T_{ki}^{\circ} = T_{ki}^{\bullet} + 1.5\frac{M_{ik}^{\bullet}}{l_{ik}}$$



$$\boldsymbol{K}\boldsymbol{u}_{\mathsf{kruto}} = \boldsymbol{0}$$
$$\boldsymbol{K}\boldsymbol{u}_{\mathsf{def.}} = \boldsymbol{r}$$

$$\begin{bmatrix} \boldsymbol{K}_{uu} & \boldsymbol{K}_{ur} \\ \boldsymbol{K}_{ru} & \boldsymbol{K}_{rr} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_u \\ \boldsymbol{u}_r \end{bmatrix} = \begin{bmatrix} \boldsymbol{r}_u \\ \boldsymbol{r}_r \end{bmatrix}$$

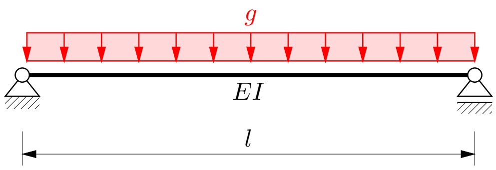
$$\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u} > 0, \quad (\boldsymbol{u} \neq \boldsymbol{0})$$

$$\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u} \geq 0, \quad (\boldsymbol{u} \neq \boldsymbol{0}) \qquad \boldsymbol{u}_u^T\boldsymbol{K}_{uu}\boldsymbol{u}_u > 0, \quad (\boldsymbol{u}_u \neq \boldsymbol{0})$$

Slika 5.3: a) Deformaciona energija se može proračunati kao $U = \frac{1}{2}\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u}$. Ako nema deformacije nema ni deformacione energije pa je za nenulte vektore $\boldsymbol{u}$ (za nula vektor trivijalno) koji predstavljaju pomjeranje konstrukcije kao kruto tijelo $u^T\boldsymbol{K}u = 0$; za vektore koji predstavljaju deformisanje konstrukcije je $\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u} > 0$; Dakle za matricu $\boldsymbol{K}$ postoje nenulti vektori u za koje je kvadratna forma $u^T\boldsymbol{K}u = 0$ i nenulti vektori $\boldsymbol{u}$ za koje je kvadratna forma pozitivna $\boldsymbol{u}^T\boldsymbol{K}\boldsymbol{u}$ što je po definiciji pozitivno-semidefinitna matrica, a takav sistem jednačina nema jedinstveno rješenje.
b) Uobičajena procedura rješavanja sistema jednačina: uvrštavanje poznatih pomjeranja $\boldsymbol{u}_r$ i rješavanje po nepoznatim pomjeranjima $\boldsymbol{u}_u$. Za svaki nenulti vektor $\boldsymbol{u}_u$ je $\boldsymbol{u}_u^T\boldsymbol{K}_{uu}\boldsymbol{u}_u > 0$ pošto dio konstrukcije koji odgovara $\boldsymbol{K}_{uu}$ ne može biti pomjeren bez deformisanja. Po definiciji pozitivno definitne matrice, matrica $\boldsymbol{K}_{uu}$ je pozitivno definitna. Determinanta pozitivno definitne matrice je uvijek pozitivna, što znači da matrica $\boldsymbol{K}_{uu}$ nije singularna što dalje znači da sistem $\boldsymbol{K}_{uu}\boldsymbol{u}_u = \boldsymbol{f}_u$ ima jedinstveno rješenje
c) Druga mogućnost formiranja rješivog sistema jednačina konstrukcije je apliciranje opruga krutosti k. Ovdje se ne radi o opruzi sa dva stepena slobode kretanja (iako ovdje možemo zamisliti i takav element, a onda uvršteno poznato pomjeranje 0 i lokalno statički kondenzovan pa dobijen element sa jednim stepenom slobode kretanja) već o opruzi sa jednim stepenom slobode kretanja čija je jednačuna ravnoteže $ku = f$ gdje su k, u i f skalari. k se ovdje dodaje globalnoj matrici krutosti, naravno, samo na dijagonalni element ${}_{Glob}K_{SSKO,SSKO}$ gdje je $SSKO$ stepen slobode kretanja u kojem je aplicirana opruga. Apliciranjem opruga se spriječi pomjeranje konstrukcije kao kruto tijelo i dobije pozitivno definitna matrica sistema jednačina.

5.3 Zadaci

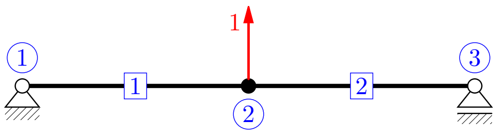
Zadatak 1 Proračunati ugib u sredini grede na slici 5.5 metodom deformacija. Pretpostaviti da je greda aksijalno kruta.



Slika 5.5

Rješenje U sredini grede ćemo umetnuti čvor. Matrica krutosti štapa zglobno vezanog na jednom kraju koji ima dva stepena slobode kretanja, koja su pomjeranja čvorova okomito na štap je

$$\frac{3EI}{l^3}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$



Slika 5.6: Obilježavanje čvorova, štapova i stepeni slobode kretanja.

Matrice krutosti štapova sa obilježenim globalnim stepenima slobode kretanja:

$$\boxed{1}\quad \begin{array}{c} \\ 0 \\ 1 \end{array}\begin{array}{c} \begin{array}{cc} 0 & \quad\quad 1 \end{array} \\ \begin{bmatrix} \frac{3EI}{(l/2)^3} & -\frac{3EI}{(l/2)^3} \\ -\frac{3EI}{(l/2)^3} & \frac{3EI}{(l/2)^3} \end{bmatrix} \end{array} \qquad \boxed{2}\quad \begin{array}{c} \\ 1 \\ 0 \end{array}\begin{array}{c} \begin{array}{cc} 1 & \quad\quad 0 \end{array} \\ \begin{bmatrix} \frac{3EI}{(l/2)^3} & -\frac{3EI}{(l/2)^3} \\ -\frac{3EI}{(l/2)^3} & \frac{3EI}{(l/2)^3} \end{bmatrix} \end{array}$$

$$K_{11} = \frac{3EI}{(l/2)^3} + \frac{3EI}{(l/2)^3} = \frac{48EI}{l^3}$$

Opterećenje

$$T^{\circ}_{ki} = T^{\bullet}_{ki} + 1.5\frac{M^{\bullet}_{ik}}{l_{ik}} = -\frac{gl}{2\cdot 2} + 1.5\frac{-g(l/2)^2}{12\cdot l/2} = -\frac{5gl}{16}$$

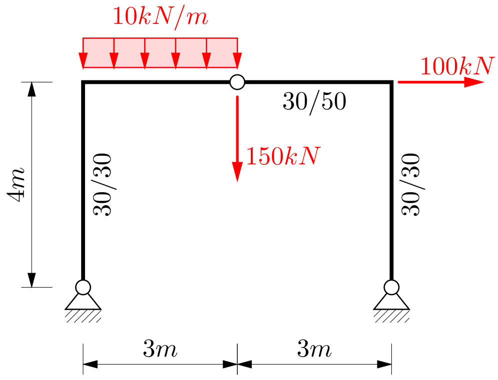
$$T^{\circ}_{ik} = T^{\bullet}_{ik} - 1.5\frac{M^{\bullet}_{ki}}{l_{ik}} = -\frac{gl}{2\cdot 2} - 1.5\frac{g(l/2)^2}{12\cdot l/2} = -\frac{5gl}{16}$$

$$f_1 = -\frac{5gl}{16} - \frac{5gl}{16} = -\frac{5gl}{8}$$

Postavljamo jednačinu metode deformacija i rješavamo

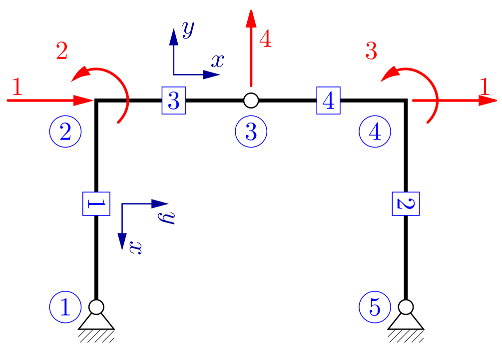
$$K_{11}u_1 = f_1 \Rightarrow \frac{48EI}{l^3}u_1 = -\frac{5gl}{8} \Rightarrow u_1 = -\frac{5gl^4}{384EI}$$

Zadatak 2 Metodom deformacija odrediti pomjeranja i presječne sile za konstrukciju na slici 5.7.
$E = 2 \cdot 10^7 kN/m^2$. Pretpostaviti da su svi štapovi aksijalno kruti.



Slika 5.7

Rješenje



Slika 5.8: Obilježavanje čvorova, štapova i stepena slobode kretanja. Pošto se pretpostavlja aksijalna krutost štapova, čvorovi 2 i 4 imaju isti SSK "1" u horizontalnom pravcu, a mogućnost vertikalnog pomjeranja čvorova 2 i 4 ne postoji

$$\begin{array}{cc}
\boxed{1} & \begin{matrix} \quad 1 \quad & \quad 2 \quad & \quad 0 \quad & 0 \end{matrix} \\
\begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} &
\begin{bmatrix}
\frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\
\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\
-\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\qquad
\begin{array}{cc}
\boxed{2} & \begin{matrix} \quad 1 \quad & \quad 3 \quad & \quad 0 \quad & 0 \end{matrix} \\
\begin{matrix} 1 \\ 3 \\ 0 \\ 0 \end{matrix} &
\begin{bmatrix}
\frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\
\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\
-\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$

$$\begin{array}{cc}
\boxed{3} & \begin{matrix} \quad 0 \quad & \quad 2 \quad & \quad 4 \quad & 0 \end{matrix} \\
\begin{matrix} 0 \\ 2 \\ 4 \\ 0 \end{matrix} &
\begin{bmatrix}
\frac{3EI_g}{l_g^3} & \frac{3EI_g}{l_g^2} & -\frac{3EI_g}{l_g^3} & 0 \\
\frac{3EI_g}{l_g^2} & \frac{3EI_g}{l_g} & -\frac{3EI_g}{l_g^2} & 0 \\
-\frac{3EI_g}{l_g^3} & -\frac{3EI_g}{l_g^2} & \frac{3EI_g}{l_g^3} & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\end{array}
\qquad
\begin{array}{cc}
\boxed{4} & \begin{matrix} \quad 4 \quad & 0 & \quad 0 \quad & \quad 3 \end{matrix} \\
\begin{matrix} 4 \\ 0 \\ 0 \\ 3 \end{matrix} &
\begin{bmatrix}
\frac{3EI_g}{l_g^3} & 0 & -\frac{3EI_g}{l_g^3} & \frac{3EI_g}{l_g^2} \\
0 & 0 & 0 & 0 \\
-\frac{3EI_g}{l_g^3} & 0 & \frac{3EI_g}{l_g^3} & -\frac{3EI_g}{l_g^2} \\
\frac{3EI_g}{l_g^2} & 0 & -\frac{3EI_g}{l_g^2} & \frac{3EI_g}{l_g}
\end{bmatrix}
\end{array}$$

$$K_{1,1} = \frac{3EI_s}{l_s^3} + \frac{3EI_s}{l_s^3} = 1265.625$$
$$K_{1,2} = \frac{3EI_s}{l_s^2} = 2531.25$$
$$K_{1,3} = \frac{3EI_s}{l_s^2} = 2531.25$$
$$K_{2,2} = \frac{3EI_s}{l_s} + \frac{3EI_g}{l_g} = 72625.00$$
$$K_{2,4} = \frac{-3EI_g}{l_g^2} = -20833.333$$
$$K_{3,3} = \frac{3EI_s}{l_s} + \frac{3EI_g}{l_g} = 72625.00$$
$$K_{3,4} = \frac{3EI_g}{l_g^2} = 20833.333$$
$$K_{4,4} = \frac{3EI_g}{l_g^3} + \frac{3EI_g}{l_g^3} = 13888.889$$

Matrica krutosti

$$\boldsymbol{K} = \begin{bmatrix} 1265.625 & 2531.250 & 2531.250 & 0 \\ & 72625.000 & 0 & -20833.333 \\ & & 72625.000 & 20833.333 \\ & \text{simetricno} & & 13888.889 \end{bmatrix}$$

Vektor sila
Za kruto vezan štap 3

$$M_{23}^{\bullet} = -\frac{q \cdot l^2}{12} = -\frac{10 \cdot 3^2}{12} = -7.5$$
$$M_{32}^{\bullet} = \frac{q \cdot l^2}{12} = \frac{10 \cdot 3^2}{12} = 7.5$$
$$V_{23}^{\bullet} = -\frac{q \cdot l}{2} = -\frac{10 \cdot 3}{2} = -15.0$$
$$V_{32}^{\bullet} = -\frac{q \cdot l}{2} = -\frac{10 \cdot 3}{2} = -15.0$$

Za štap 3 zglobno vezan u čvoru 3

$$M_{23}^{\circ} = M_{23}^{\bullet} - 0.5 \cdot M_{32}^{\bullet} = -7.5 - 0.5 \cdot 7.5 = -11.25$$
$$M_{32}^{\circ} = 0$$
$$V_{23}^{\circ} = V_{23}^{\bullet} - 1.5 \frac{M_{32}^{\bullet}}{l_{23}} = -15 - 1.5 \frac{7.5}{3} = -18.75$$
$$V_{32}^{\circ} = V_{32}^{\bullet} + 1.5 \frac{M_{32}^{\bullet}}{l_{23}} = -15 + 1.5 \frac{7.5}{3} = -11.25$$

$$\boldsymbol{f} = \boldsymbol{f}^3 + \boldsymbol{f}_n = \begin{bmatrix} 0.00 \\ -11.25 \\ 0.00 \\ -11.25 \end{bmatrix} + \begin{bmatrix} 100.00 \\ 0.00 \\ 0.00 \\ -150.00 \end{bmatrix} = \begin{bmatrix} 100.00 \\ -11.25 \\ 0.00 \\ -161.25 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} 0.092172 \\ -0.027734 \\ 0.021154 \\ -0.084943 \end{bmatrix}$$

Proračun rotacija krajeva štapa u čvoru 3

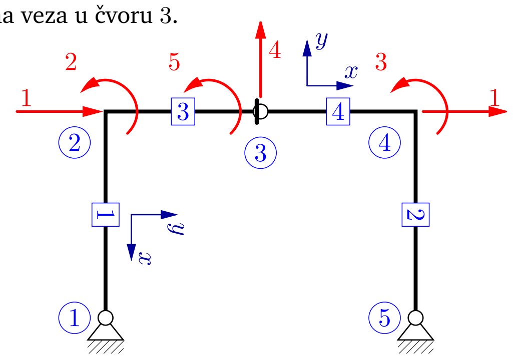
$$\varphi_{32}^{\circ} = -0.5 \cdot \varphi_{23} + 1.5\frac{\Delta_{23}}{l_{23}} + 0.25 \cdot \frac{M_{32}^{\bullet}}{k_{23}}$$
$$\varphi_{34}^{\circ} = -0.5 \cdot \varphi_{43} + 1.5\frac{\Delta_{34}}{l_{34}} + 0.25 \cdot \frac{M_{34}^{\bullet}}{k_{34}}$$

$$k_{23} = \frac{EI}{l_{23}} = \frac{2 \cdot 10^7 \cdot 0.003125}{3} = 20833.333$$
$$k_{34} = \frac{EI}{l_{34}} = \frac{2 \cdot 10^7 \cdot 0.003125}{3} = 20833.333$$
$$\Delta_{23} = v_k - v_i = -0.084943 - 0 = -0.084943$$
$$\Delta_{34} = v_k - v_i = 0 - (-0.084943) = 0.084943$$

$$\varphi_{32}^{\circ} = -0.5 \cdot (-0.027734) + 1.5\frac{-0.084943}{3} + 0.25 \cdot \frac{7.5}{20833.333} = -0.028514 \tag{5.4}$$
$$\varphi_{34}^{\circ} = -0.5 \cdot 0.021154 + 1.5\frac{0.084943}{3} + 0.25 \cdot \frac{0}{k_{34}} = 0.031894 \tag{5.5}$$

Uvođenje stepena slobode kretanja u proračun

Sad ćemo, da bismo vježbali metodu deformacija, uvesti stepen slobode kretanja 5 u proračun, kako je prikazano na slici 5.9. U ovom slučaju štap 3 je kruto vezan na oba kraja, dok je štap 4 štap sa otpuštenim momentom kod čvora 3 čime se ostvaruje zglobna veza u čvoru 3.



Slika 5.9

$$\boxed{1}\quad \begin{array}{c|cccc} & 1 & 2 & 0 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 2 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 0 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{2}\quad \begin{array}{c|cccc} & 1 & 3 & 0 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 3 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 0 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{3}\quad \begin{array}{c|cccc} & 0 & 2 & 4 & 5 \\ \hline 0 & \frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} & -\frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} \\ 2 & \frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} \\ 4 & -\frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} & \frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} \\ 5 & \frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} \end{array}$$

$$\boxed{4}\quad \begin{array}{c|cccc} & 4 & 0 & 0 & 3 \\ \hline 4 & \frac{3EI_g}{l_g^3} & 0 & -\frac{3EI_g}{l_g^3} & \frac{3EI_g}{l_g^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3EI_g}{l_g^3} & 0 & \frac{3EI_g}{l_g^3} & -\frac{3EI_g}{l_g^2} \\ 3 & \frac{3EI_g}{l_g^2} & 0 & -\frac{3EI_g}{l_g^2} & \frac{3EI_g}{l_g} \end{array}$$

$$K_{1,1} = \frac{3EI_s}{l_s^3} + \frac{3EI_s}{l_s^3} = 1265.625$$
$$K_{1,2} = \frac{3EI_s}{l_s^2} = 2531.25$$
$$K_{1,3} = \frac{3EI_s}{l_s^2} = 2531.25$$
$$K_{2,2} = \frac{3EI_s}{l_s} + \frac{4EI_g}{l_g} = 93458.333$$
$$K_{2,4} = -\frac{6EI_g}{l_g^2} = -41666.667$$
$$K_{2,5} = \frac{2EI_g}{l_g} = 41666.667$$
$$K_{3,3} = \frac{3EI_s}{l_s} + \frac{3EI_g}{l_g} = 72625.00$$
$$K_{3,4} = \frac{3EI_g}{l_g^2} = 20833.333$$
$$K_{4,4} = \frac{12EI_g}{l_g^3} + \frac{3EI_g}{l_g^3} = 34722.222$$
$$K_{4,5} = -\frac{6EI_g}{l_g^2} = -41666.667$$
$$K_{5,5} = \frac{4EI_g}{l_g} = 83333.333$$

Matrica krutosti

$$\boldsymbol{K} = \begin{bmatrix} 1265.625 & 2531.250 & 2531.250 & 0 & 0 \\ & 93458.333 & 0 & -41666.667 & 41666.667 \\ & & 72625.000 & 20833.333 & 0 \\ & \text{simetricno} & & 34722.222 & -41666.667 \\ & & & & 83333.333 \end{bmatrix}$$

Vektor sila

Za kruto vezan štap 3

$$M_{23}^{\bullet} = -\frac{q \cdot l^2}{12} = -\frac{10 \cdot 3^2}{12} = -7.5$$
$$M_{32}^{\bullet} = \frac{q \cdot l^2}{12} = \frac{10 \cdot 3^2}{12} = 7.5$$
$$V_{23}^{\bullet} = -\frac{q \cdot l}{2} = -\frac{10 \cdot 3}{2} = -15.0$$
$$V_{32}^{\bullet} = -\frac{q \cdot l}{2} = -\frac{10 \cdot 3}{2} = -15.0$$

Globalni vektor sila

$$\boldsymbol{f} = \begin{bmatrix} 100.00 \\ -7.50 \\ 0.00 \\ -165.00 \\ 7.50 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{Ku} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} 0.092172 \\ -0.027734 \\ 0.021154 \\ -0.084943 \\ -0.028514 \end{bmatrix}$$

Zaključujemo da je

$$\boldsymbol{u}_5 = \varphi_{32}^{\circ}$$

što je proračunato jednačinom 5.4.

Proračun rotacije kraja štapa 3 u čvoru 3:

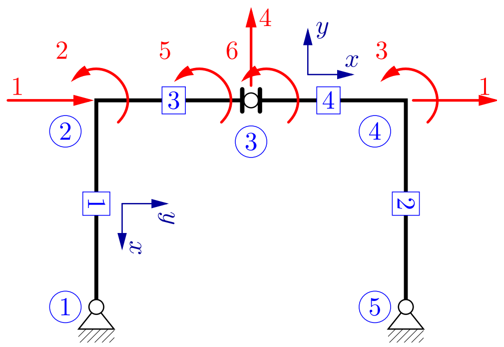
$$\Delta_{34} = v_k - v_i = 0 - (-0.084943) = 0.084943 \tag{5.6}$$

$$\varphi^{\circ}_{34} = -0.5 \cdot \varphi_{43} + 1.5\frac{\Delta_{34}}{l_{34}} + 0.25 \cdot \frac{M^{\bullet}_{34}}{k_{34}}$$

$$\varphi^{\circ}_{34} = -0.5 \cdot 0.021154 + 1.5\frac{0.084943}{3} + 0.25 \cdot \frac{0}{k_{34}} = 0.031894 \tag{5.7}$$

Uvođenje još jednog stepena slobode kretanja u proračun

Sad ćemo, ponovo u svrhu vježbanja metode deformacija, uvesti stepen slobode kretanja 6 u proračun, kako je prikazano na slici 5.10. Štapovi 3 i 4 su kruto vezani na oba kraja, ali imaju različite rotacione stepene slobode kretanja u čvoru 3 i isti vertikalni (4) SSK čime se ostvaruje zglobna veza.



Slika 5.10

$$\boxed{1}\quad \begin{array}{c|cccc} & 1 & 2 & 0 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 2 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 0 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{2}\quad \begin{array}{c|cccc} & 1 & 3 & 0 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 3 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 0 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{3}\quad \begin{array}{c|cccc} & 0 & 2 & 4 & 5 \\ \hline 0 & \frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} & -\frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} \\ 2 & \frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} \\ 4 & -\frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} & \frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} \\ 5 & \frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} \end{array}$$

$$\boxed{4}\quad \begin{array}{c|cccc} & 4 & 6 & 0 & 3 \\ \hline 4 & \frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} & -\frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} \\ 6 & \frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} \\ 0 & -\frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} & \frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} \\ 3 & \frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} \end{array}$$

$$K_{1,1} = \frac{3EI_s}{l_s^3} + \frac{3EI_s}{l_s^3} = 1265.625$$

$$K_{1,2} = \frac{3EI_s}{l_s^2} = 2531.25$$

$$K_{1,3} = \frac{3EI_s}{l_s^2} = 2531.25$$

$$K_{2,2} = \frac{3EI_s}{l_s} + \frac{4EI_g}{l_g} = 93458.333$$

$$K_{2,4} = -\frac{6EI_g}{l_g^2} = -41666.667$$

$$K_{2,5} = \frac{2EI_g}{l_g} = 41666.667$$

$$K_{3,3} = \frac{3EI_s}{l_s} + \frac{4EI_g}{l_g} = 93458.333$$

$$K_{3,4} = \frac{6EI_g}{l_g^2} = 41666.667$$

$$K_{3,6} = \frac{2EI_g}{l_g} = 41666.667$$

$$K_{4,4} = \frac{12EI_g}{l_g^3} + \frac{12EI_g}{l_g^3} = 55555.556$$

$$K_{4,5} = -\frac{6EI_g}{l_g^2} = -41666.667$$

$$K_{4,6} = \frac{6EI_g}{l_g^2} = 41666.667$$

$$K_{5,5} = \frac{4EI_g}{l_g} = 83333.333$$

$$K_{6,6} = \frac{4EI_g}{l_g} = 83333.333$$

Matrica krutosti

$$\boldsymbol{K} = \begin{bmatrix} 1265.625 & 2531.250 & 2531.250 & 0 & 0 & 0 \\ & 93458.333 & 0 & -41666.667 & 41666.667 & 0 \\ & & 93458.333 & 41666.667 & 0 & 41666.667 \\ & & & 55555.556 & -41666.667 & 41666.667 \\ & \text{simetricno} & & & 83333.333 & 0 \\ & & & & & 83333.333 \end{bmatrix}$$

Globalni vektor sila

$$\boldsymbol{f} = \begin{bmatrix} 100.00 \\ -7.50 \\ 0.00 \\ -165.00 \\ 7.50 \\ 0.00 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} 0.092172 \\ -0.027734 \\ 0.021154 \\ -0.084943 \\ -0.028514 \\ 0.031894 \end{bmatrix}$$

Možemo zaključiti da je

$$\boldsymbol{u}_6 = \varphi^{\circ}_{34}$$

proračunato jednačinama 5.5 i 5.7

Proračun presječnih sila na krajevima štapova

Presječne sile na krajevima štapa proračunavamo sa

$$\boldsymbol{f}^e = \boldsymbol{K}^e \boldsymbol{u}^e - \boldsymbol{s}_e^{\bullet}$$

U razvijenom obliku možemo pisati

Za kruto vezan štap

$$M_{ik} = k_{ik}\left(4\varphi_i + 2\varphi_k - 6\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\bullet}$$
$$M_{ki} = k_{ik}\left(2\varphi_i + 4\varphi_k - 6\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ki}^{\bullet}$$
$$T_{ik} = \bar{k}_{ik}\left(6\varphi_i + 6\varphi_k - 12\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\bullet}$$
$$T_{ki} = \bar{k}_{ik}\left(-6\varphi_i - 6\varphi_k + 12\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\bullet}$$

gdje je $k_{ik} = \dfrac{EI}{l_{ik}}$ i $\bar{k}_{ik} = \dfrac{EI}{l_{ik}^2}$

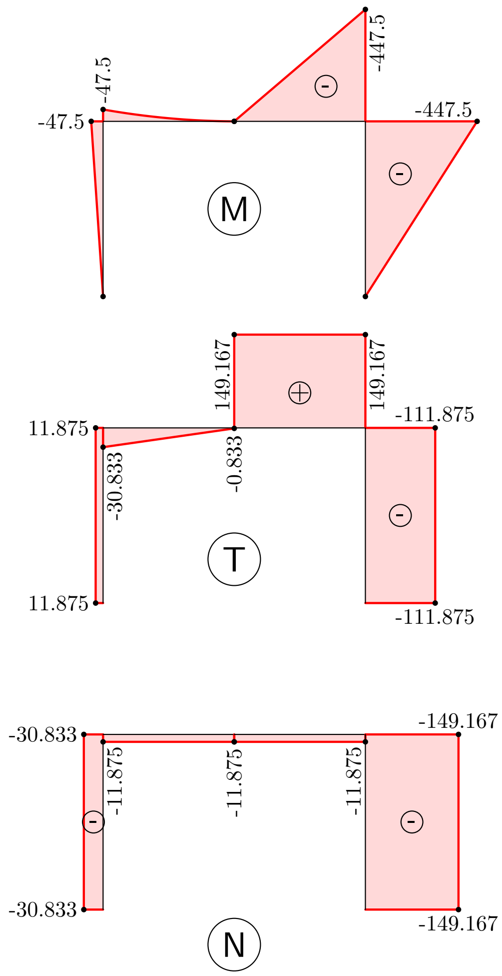
Za štap kruto vezan u čvoru i, a zglobno u čvoru k

$$M_{ik} = k_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$
$$M_{ki} = 0$$
$$T_{ik} = \bar{k}_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$
$$T_{ki} = -\bar{k}_{ik}\left(3\varphi_i - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

Štap zglobno vezan u čvoru i, a kruto u čvoru k

$$M_{ik} = 0$$
$$M_{ki} = k_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$
$$T_{ik} = \bar{k}_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$
$$T_{ki} = -\bar{k}_{ik}\left(3\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

Na stranici 102 dat je numerički proračun presječnih sila na krajevima štapova.



Slika 5.11: Dijagrami presječnih sila

Štap 1

$$M_{21} = 3375.0 \cdot \left(3 \cdot (-0.027734) - 3\frac{-0.092172}{4}\right) = -47.5$$
$$M_{12} = 0$$
$$T_{21} = 843.75 \left(3 \cdot (-0.027734) - 3\frac{-0.092172}{4}\right) = -11.875$$
$$T_{12} = -843.75 \left(3 \cdot (-0.027734) - 3\frac{-0.092172}{4}\right) = 11.875$$

Štap 2

$$M_{23} = 20833.333 \left(4 \cdot (-0.027734) + 2 \cdot (-0.028514) - 6\frac{-0.084943}{3}\right) - (-7.5) = 47.50$$
$$M_{32} = 20833.333 \left(2 \cdot (-0.027734) + 4 \cdot (-0.028514) - 6\frac{-0.084943}{3}\right) - 7.5 = 0.00$$
$$T_{23} = 6944.444 \left(6 \cdot (-0.027734) + 6 \cdot (-0.028514) - 12\frac{-0.084943}{3}\right) - (-15.0) = 30.833$$
$$T_{32} = 6944.444 \left(-6 \cdot (-0.027734) - 6 \cdot (-0.028514) + 12\frac{-0.084943}{3}\right) - (-15.0) = -0.833$$

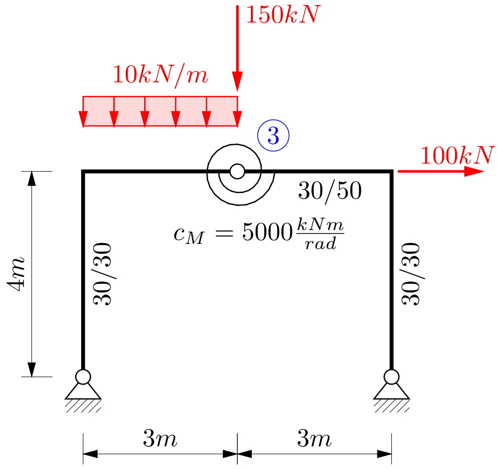
Štap 3

$$M_{34} = 20833.333 \left(4 \cdot 0.031894 + 2 \cdot 0.021154 - 6\frac{-(-0.084943)}{3}\right) = 0.00$$
$$M_{43} = 20833.333 \left(2 \cdot 0.031894 + 4 \cdot 0.021154 - 6\frac{-(-0.084943)}{3}\right) = -447.50$$
$$T_{34} = 6944.444 \left(6 \cdot 0.031894 + 6 \cdot 0.021154 - 12\frac{-(-0.084943)}{3}\right) = -149.167$$
$$T_{43} = 6944.444 \left(-6 \cdot 0.031894 - 6 \cdot 0.021154 + 12\frac{-(-0.084943)}{3}\right) = 149.167$$

Štap 4

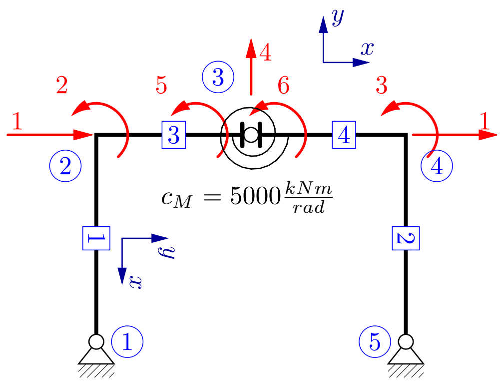
$$M_{45} = 3375.0 \cdot \left(3 \cdot 0.021154 - 3\frac{-0.092172}{4}\right) = 447.50$$
$$M_{54} = 0$$
$$T_{45} = 843.75 \left(3 \cdot 0.021154 - 3\frac{-0.092172}{4}\right) = 111.875$$
$$T_{54} = -843.75 \left(3 \cdot 0.021154 - 3\frac{-0.092172}{4}\right) = -111.875$$

Zadatak 3 Zglob u čvoru 3 na konstrukciji iz prethodnog zadatka ukrutiti vezom krutosti $c_M = 5000 kNm/rad$ kako je prikazano na slici 5.12. Proračunati pomjeranja čvorova, presječne sile i nacrtati dijagrame presječnih sila.
$E = 2 \cdot 10^7 kN/m^2$.



Slika 5.12

Rješenje Obilježićemo stepene slobode kretanja kao u prethodnom zadatku



Slika 5.13

Matrica krutosti opruge je $\begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$ Sa slike 5.13 vidimo da opruga povezuje SSK 5 i 6 pa ćemo sljedeću matricu dodati matrici krutosti konstrukcije koju smo proračunali u prethodnom zadatku

$$\begin{array}{cc} & \begin{array}{cc} 5 & 6 \end{array} \\ \begin{array}{c} 5 \\ 6 \end{array} & \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \end{array}$$

pa dobijamo

$$\boldsymbol{K} =$$

$$\begin{bmatrix} 1265.625 & 2531.250 & 2531.250 & 0 & 0 & 0 \\ & 93458.333 & 0 & -41666.667 & 41666.667 & 0 \\ & & 93458.333 & 41666.667 & 0 & 41666.667 \\ & & & 55555.556 & -41666.667 & 41666.667 \\ & \text{simetricno} & & & 83333.333 + 5000.000 & 0 - 5000.000 \\ & & & & & 83333.333 + 5000.000 \end{bmatrix}$$

Globalni vektor sila ostaje isti kao u prethodnom zadatku

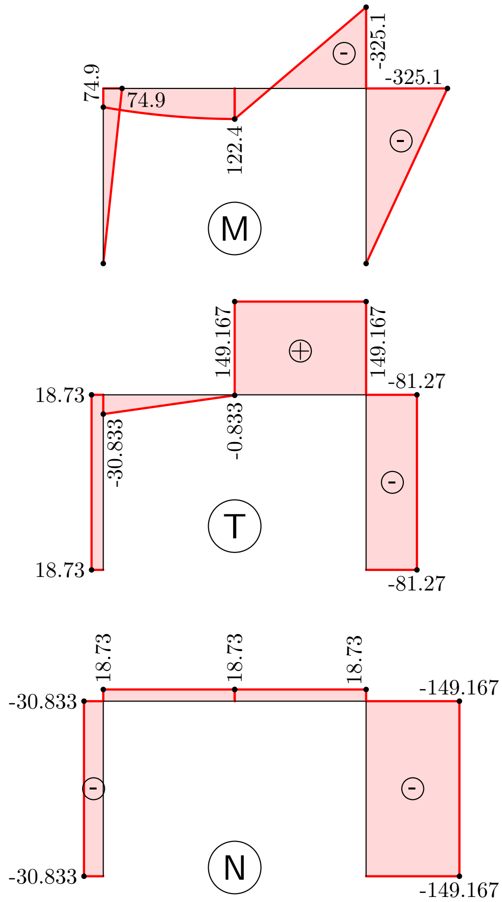
$$\boldsymbol{f} = \begin{bmatrix} 100.00 \\ -7.50 \\ 0.00 \\ -165.00 \\ 7.50 \\ 0.00 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} 0.092172 \\ -0.015645 \\ 0.009065 \\ -0.039863 \\ -0.010550 \\ 0.013930 \end{bmatrix}$$

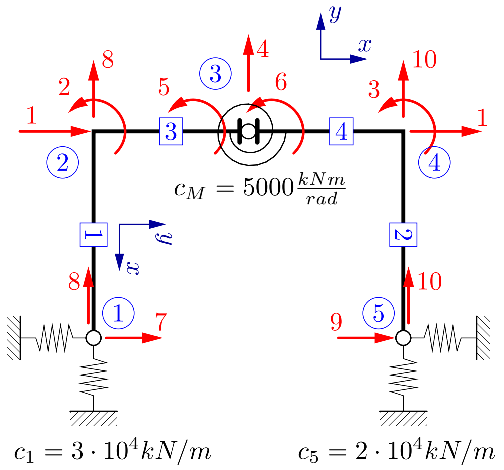
Sila u opruzi

$$\boldsymbol{f}^{opr} = \begin{bmatrix} 5000 & -5000 \\ -5000 & 5000 \end{bmatrix} \cdot \begin{bmatrix} -0.010550 \\ 0.013930 \end{bmatrix} = \begin{bmatrix} -122.401 \\ 122.401 \end{bmatrix}$$



Slika 5.14: Dijagrami presječnih sila

Zadatak 4 Konstrukciju iz prethodnog zadatka proračunati sa elastičnim osloncima umjesto krutih, prema slici 5.15.



Slika 5.15

Rješenje Na slici 5.15 su također dati stepeni slobode kretanja. Možemo primijetiti da smo u čvorovima 1 i 5 uveli dodatne stepene slobode kretanja 7, 8 i 9, 10. Uvodeći stepen slobode kretanja 8 u čvoru 1, a zadržavajući pretpostavku o aksijalnoj krutost štapova, čvor 2 takođe dobija mogućnost pomjeranja 8. Isto važi i za čvor 4 i SSK 10.

$$\boxed{1}\quad \begin{array}{c|cccc} & 1 & 2 & 7 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 2 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 7 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{2}\quad \begin{array}{c|cccc} & 1 & 3 & 9 & 0 \\ \hline 1 & \frac{3EI_s}{l_s^3} & \frac{3EI_s}{l_s^2} & -\frac{3EI_s}{l_s^3} & 0 \\ 3 & \frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s} & -\frac{3EI_s}{l_s^2} & 0 \\ 9 & -\frac{3EI_s}{l_s^3} & -\frac{3EI_s}{l_s^2} & \frac{3EI_s}{l_s^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$\boxed{3}\quad \begin{array}{c|cccc} & 8 & 2 & 4 & 5 \\ \hline 8 & \frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} & -\frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} \\ 2 & \frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} \\ 4 & -\frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} & \frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} \\ 5 & \frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} \end{array}$$

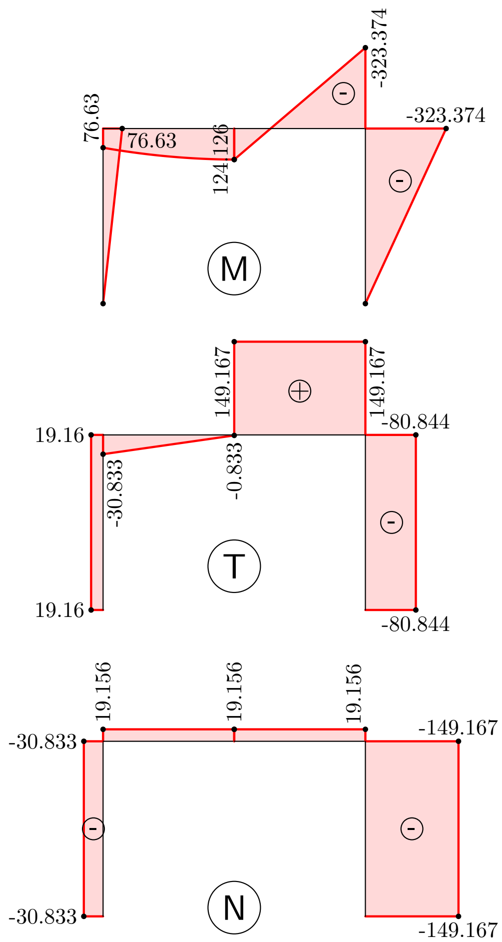
$$\boxed{4}\quad \begin{array}{c|cccc} & 4 & 6 & 10 & 3 \\ \hline 4 & \frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} & -\frac{12EI_g}{l_g^3} & \frac{6EI_g}{l_g^2} \\ 6 & \frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} \\ 10 & -\frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} & \frac{12EI_g}{l_g^3} & -\frac{6EI_g}{l_g^2} \\ 3 & \frac{6EI_g}{l_g^2} & \frac{2EI_g}{l_g} & -\frac{6EI_g}{l_g^2} & \frac{4EI_g}{l_g} \end{array}$$

Vektor sila

$$\boldsymbol{f} = \begin{bmatrix} 100.00 \\ -7.50 \\ 0 \\ -165.00 \\ 7.50 \\ 0 \\ 0 \\ -15.00 \\ 0 \\ 0 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{Ku} = \boldsymbol{f}$ dobijamo

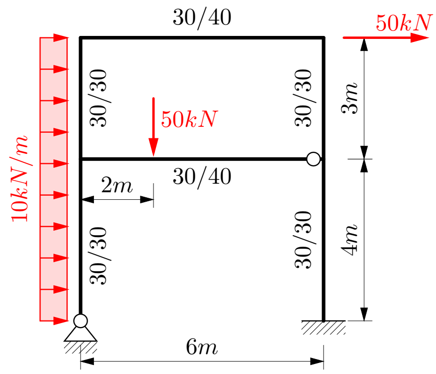
$$\boldsymbol{u} = \begin{bmatrix} 0.098800 \\ -0.016972 \\ 0.008249 \\ -0.044748 \\ -0.011794 \\ 0.013031 \\ 0.000639 \\ -0.001028 \\ 0.004042 \\ -0.007458 \end{bmatrix}$$



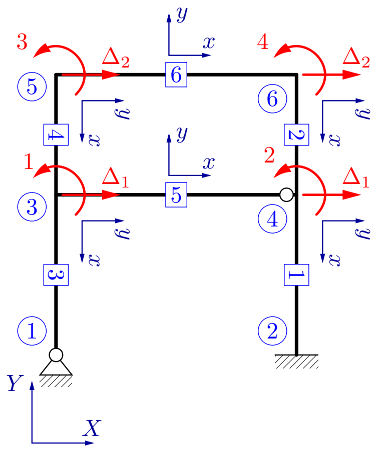
Slika 5.16: Dijagrami presječnih sila

$$
\begin{bmatrix}
1265.625 & 2531.250 & 2531.250 & 0 & 0 & 0 & -632.813 & 0 & -632.813 & 0 \\
 & 93458.333 & 0 & -41666.667 & 41666.667 & 0 & -2531.250 & 41666.667 & 0 & 0 \\
 & & 93458.333 & 41666.667 & 0 & 41666.667 & 0 & 0 & -2531.250 & -41666.667 \\
 & & & 55555.556 & -41666.667 & 41666.667 & 0 & -27777.778 & 0 & -27777.778 \\
 & & & & 88333.333 & -5000.000 & 0 & 41666.667 & 0 & 0 \\
 & & & & & 88333.333 & 0 & 0 & 0 & -41666.667 \\
 & & & & & & 30632.813 & 0 & 0 & 0 \\
 & \text{simetricno} & & & & & & 57777.778 & 0 & 0 \\
 & & & & & & & & 20632.813 & 0 \\
 & & & & & & & & & 47777.778
\end{bmatrix}
$$

Zadatak 5 Metodom deformacija odrediti pomjeranja i presječne sile za konstrukciju na slici 5.17



Slika 5.17



Slika 5.18: Obilježavanje stepeni slobode kretanja

Rješenje

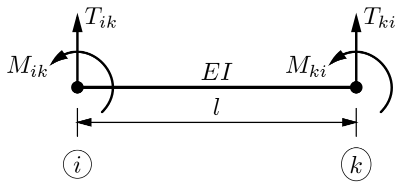
Postavljanjem Takabejevih jednačina

Pomjeranja koja se traže

- Rotacije čvorova $\varphi_3, \varphi_4, \varphi_5, \varphi_6$
- Translatorna pomjeranja Δ_1, Δ_2

Takabejeve jednačine

Štap kruto vezan na oba kraja



Slika 5.19

$$M_{ik} = k_{ik}\left(2\varphi_i + \varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\bullet}$$

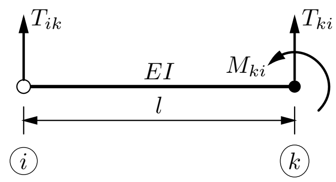
$$M_{ki} = k_{ik}\left(\varphi_i + 2\varphi_k - 3\frac{\Delta_{ik}}{l_{ik}}\right) - M_{ki}^{\bullet}$$

$$T_{ik} = \bar{k}_{ik}\left(\varphi_i + \varphi_k - 2\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\bullet}$$

$$T_{ki} = \bar{k}_{ik}\left(-\varphi_i - \varphi_k + 2\frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\bullet}$$

gdje je $k_{ik} = \frac{2EI}{l_{ik}}, \bar{k}_{ik} = \frac{6EI}{l_{ik}^2}, \bar{k}_{ik} = \frac{3k_{ik}}{l_{ik}}$

Štap sa oslobođenim momentom na kod čvora i



Slika 5.20

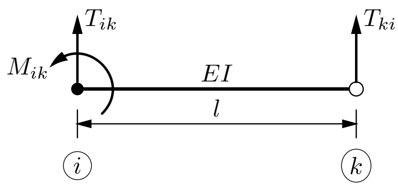
$$M_{ik} = 0$$

$$M_{ki} = 1.5k_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - M_{ki}^{\circ}$$

$$T_{ik} = 0.5\bar{k}_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$

$$T_{ki} = -0.5\bar{k}_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

Štap sa oslobođenim momentom na kod čvora k



Slika 5.21

$$M_{ik} = 1.5k_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$

$$M_{ki} = 0$$

$$T_{ik} = 0.5\bar{k}_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$

$$T_{ki} = -0.5\bar{k}_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

Postavljanje Takabejevih jednačina ravnoteže

Rotacija čvora 3

$$M_{31} = 1.5 \cdot 10125 \left(\varphi_3 - \frac{-\Delta_1}{4}\right) - 20$$
$$M_{34} = 1.5 \cdot 16000(\varphi_3) - (-55.555)$$
$$M_{35} = 13500\,[2\varphi_3 + \varphi_5 - (-\Delta_2) - (-7.5)]$$

$$M_{31} = 15187.5 \cdot \varphi_3 \;\; +3796.875 \cdot \Delta_1 - 20.00$$
$$M_{34} = 24000.0 \cdot \varphi_3 \qquad +55.555$$
$$M_{35} = 27000.0 \cdot \varphi_3 + 13500.000 \cdot \varphi_5 \; + 13500 \cdot \Delta_2 + 7.5$$

$$\sum_{k=1,4,5} M_{3k} = 0$$

$$66187.5 \cdot \varphi_3 + 13500 \cdot \varphi_5 + 3796.875 \cdot \Delta_1 + 13500 \cdot \Delta_2 + 43.055 = 0$$

Rotacija čvora 4

$$M_{42} = 10125\,[2\varphi_4 - 0.75(-\Delta_1)]$$
$$M_{46} = 13500\,[2\varphi_4 + \varphi_6 - (-\Delta_2)]$$

$$M_{42} = 20250 \cdot \varphi_4 \quad +7593.75 \cdot \Delta_1$$
$$M_{46} = 27000 \cdot \varphi_4 + 13500 \cdot \varphi_6 + 13500 \cdot \Delta_2$$

$$\sum_{k=2,6} M_{4k} = 0$$

$$47250 \cdot \varphi_4 + 13500 \cdot \varphi_6 + 7593.75 \cdot \Delta_1 + 13500 \cdot \Delta_2 = 0$$

Rotacija čvora 5

$$M_{53} = 13500\,[\varphi_3 + 2\varphi_5 - (-\Delta_2) - 7.5]$$
$$M_{56} = 16000\,(2\varphi_5 + \varphi_6)$$

$$M_{53} = 13500 \cdot \varphi_3 + 27000 \cdot \varphi_5 + 13500 \cdot \Delta_2 - 7.5$$
$$M_{56} = 32000 \cdot \varphi_5 + 16000 \cdot \varphi_6$$

$$\sum_{k=3,6} M_{5k} = 0$$

$$59000 \cdot \varphi_5 + 13500 \cdot \varphi_3 + 16000 \cdot \varphi_6 + 13500 \cdot \Delta_2 - 7.5 = 0$$

Rotacija čvora 6

$$M_{65} = 16000\,(\varphi_5 + 2\varphi_6)$$
$$M_{64} = 13500\,[\varphi_4 + 2\varphi_6 - (-\Delta_2)]$$

$$M_{65} = 16000 \cdot \varphi_5 + 32000 \cdot \varphi_6$$
$$M_{64} = 13500 \cdot \varphi_4 + 27000 \cdot \varphi_6 + 13500 \cdot \Delta_2$$

$$\sum_{k=4,5} M_{6k} = 0$$

$$59000 \cdot \varphi_6 + 13500 \cdot \varphi_4 + 16000 \cdot \varphi_5 + 13500 \cdot \Delta_2 = 0$$

Pomjeranje Δ_1

$$T_{31} = -0.5 \cdot 7593.75 \left(\varphi_3 - \frac{-\Delta_1}{4}\right) - (-25)$$
$$T_{42} = 7593.75 \left(-\varphi_4 + 2\frac{-\Delta_1}{4}\right)$$

$$T_{31} = -3796.875 \cdot \varphi_3 - 949.21875\Delta_1 + 25$$
$$T_{42} = -7593.750 \cdot \varphi_4 - 3796.875 \cdot \Delta_1$$

$$\sum T_I = T_{31} + T_{42} + P = 0$$

$$-3796.875 \cdot \varphi_3 - 7593.750 \cdot \varphi_4 - 4746.094\Delta_1 = -105$$

Pomjeranje Δ_2

$$T_{53} = 13500 \left(-\varphi_3 - \varphi_5 + 2\frac{-\Delta_2}{3}\right) - (-15)$$
$$T_{64} = 13500 \left(-\varphi_6 - \varphi_4 + 2\frac{-\Delta_2}{3}\right)$$

$$T_{53} = -13500 \cdot \varphi_3 - 13500 \cdot \varphi_5 - 9000\Delta_2 + 15$$
$$T_{64} = -13500 \cdot \varphi_6 - 13500 \cdot \varphi_4 - 9000\Delta_2$$

$$\sum T_{II} = T_{53} + T_{64} + P = 0$$

$$-13500 \cdot \varphi_3 - 13500 \cdot \varphi_4 - 13500 \cdot \varphi_5 - 13500 \cdot \varphi_6 - 18000\Delta_2 = -65$$

Konačno možemo napisati sistem jednačina

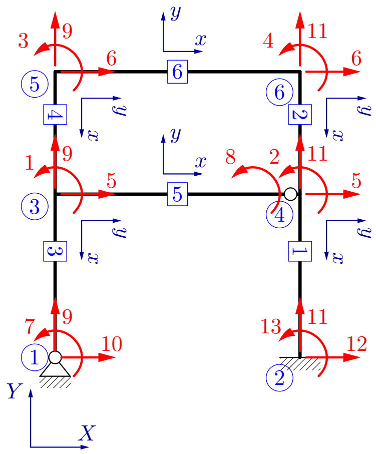
$$\begin{array}{cccccc} \varphi_3 & \varphi_4 & \varphi_5 & \varphi_6 & \Delta_1 & \Delta_2 \\ \end{array}$$
$$\begin{bmatrix} 66187.5 & 0 & 13500 & 0 & 3796.875 & 13500 \\ 0 & 47250 & 0 & 13500 & 7593.75 & 13500 \\ 13500 & 0 & 59000 & 16000 & 0 & 13500 \\ 0 & 13500 & 16000 & 59000 & 0 & 13500 \\ 3796.875 & 7593.75 & 0 & 0 & 4746.094 & 0 \\ 13500 & 13500 & 13500 & 13500 & 0 & 18000 \end{bmatrix} \begin{bmatrix} \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} -43.055 \\ 0 \\ 7.5 \\ 0 \\ 105 \\ 65 \end{bmatrix}$$

Rješenjem sistema jednačina dobijaju se pomjeranja

$$\begin{bmatrix} \varphi_3 \\ \varphi_4 \\ \varphi_5 \\ \varphi_6 \\ \Delta_1 \\ \Delta_2 \end{bmatrix} = \begin{bmatrix} -0.0079122 \\ -0.0149656 \\ -0.0032522 \\ -0.0012122 \\ 0.0523981 \\ 0.0241177 \end{bmatrix} \tag{5.8}$$

Sad ćemo istu konstrukciju riješiti koristeći Scilab. U prvom koraku implementiraćemo matrice krutosti štapova (kruto i zglobno vezan) i odgovarajuće vektore opterećenja, a zatim ćemo formirati skriptu kojom ćemo proračunati konstrukciju.

Takođe ćemo uvesti dodatne stepene slobode kretanja kako bismo dobili sva pomjeranja, reakcije i presječne sile koristeći jedinstvenu kompaktnu formu proračuna. Da pojasnimo ovo, dakle u prethodnoj sekciji smo za štapove 3 i 5 koji imaju otpušten momenat u čvoru k koristili kondenzovanu matricu krutosti pa rotacija kraja štapa φ_k kod čvora k nije figurirala u globalnom sistemu jednačina, već rotaciju kraja štapa moramo proračunati naknadno koristeći jednakost 5.3, dok ćemo u sljedećem uvesti stepene slobode kretanja 7 i 8 (vidjeti sliku 5.22) a onda štapove 3 i 5 posmatrati kao kruto vezane. Zglobna veza između štapa 5 i ostalih štapova u čvoru 4 je postignuta time što ostali štapovi imaju različit stepen slobode kretanja u čvoru 4 (SSK 2)



Slika 5.22: Obilježavanje stepeni slobode kretanja. Štapovi su aksijalno kruti.

Listing 5.1: Formiranje matrice krutosti u lokalnom koordinatnom sistemu, `Greda2D4x4.sci`

```
function rezultat = Greda2D4x4(idx, koordinate, elementi,
    presjek)
// Greda2D4x4 - Matrica krutosti ravnog grednog stapa

Em = presjek(1);
I = presjek(2);

x1 = koordinate(elementi(idx, 1), 1);
y1 = koordinate(elementi(idx, 1), 2);
x2 = koordinate(elementi(idx, 2), 1);
y2 = koordinate(elementi(idx, 2), 2);

duzina = sqrt((x2 - x1)^2 + (y2 - y1)^2);

km12 = 12 * Em * I / duzina^3;
km6= km12 * duzina / 2;
km4= km6 * duzina / 1.5;
km2= km4 / 2;

rezultat = [
 km12  km6 -km12  km6
 km6   km4 -km6   km2
-km12 -km6  km12 -km6
 km6   km2 -km6   km4];
endfunction
```

Listing 5.2: Vektor ekvivalentnog opterećenja od ravnomjerno raspodjeljenog opterećenja, `RavnomjernoOpt.sci`

```
function rezultat = RavnomjernoOpt(idx, koordinate, elementi,
    p)
// RavnomjernoOpt - Vektor ekvivalentnog opterecenja na gredi
// od ravnomjernog opterecenja

x1 = koordinate(elementi(idx, 1), 1);
y1 = koordinate(elementi(idx, 1), 2);
x2 = koordinate(elementi(idx, 2), 1);
y2 = koordinate(elementi(idx, 2), 2);

d = sqrt((x2 - x1)^2 + (y2 - y1)^2);

rezultat = [
p*d/2
p*d^2/12
p*d/2
-p*d^2/12];

endfunction
```

Listing 5.3: Vektor ekvivalentnog opterećenja od koncentrične sile okomito na gredu, `KoncOpt.sci`

```
function rezultat = KoncOpt(idx, koordinate, elementi, P, a)
// KoncOpt - Vektor ekvivalentnog opterecenja na gredi
// od koncentricne sile okomito na gredu

x1 = koordinate(elementi(idx, 1), 1);
y1 = koordinate(elementi(idx, 1), 2);
x2 = koordinate(elementi(idx, 2), 1);
y2 = koordinate(elementi(idx, 2), 2);

d = sqrt((x2 - x1)^2 + (y2 - y1)^2);
b = d - a;

rezultat = [
P*b*b*(3*a+b)/(d^3)
P*a*b*b/d^2
P*a*a*(3*b+a)/(d^3)
-P*a*a*b/d^2
];
endfunction
```

Sa ovim možemo napisati sljedeću skriptu

Listing 5.4: Proračun konstrukcije, `Zadatak5.sce`

```
// Tehnicka metoda deformacija
// Zadatak 5
// ***********************************************************
clear;
clc;
exec('./Greda2D4x4.sci', -1);
exec('./RavnomjernoOpt.sci', -1);
exec('./KoncOpt.sci', -1);

// ****************** ULAZNI PODACI ************************
// duzina m; sila kN;

//           E       I
presjekG = [3*10^7, 0.3*0.4^3/12];
presjekS = [3*10^7, 0.3^4/12];

// Koordinate cvorova
koordinate = [
0 0
6 0
0 4
6 4
0 7
6 7];

// Definisanje stapova krajnjim cvorovima
elementi =[
2 4
4 6
1 3
3 5
3 4
5 6];

SSKEL = [
5 2 12 13
6 4 5  2
5 1 10 7
6 3 5  1
9 1 11 8
9 3 11 4];

Kels = hypermat([4 4 size(elementi,'r')]);
Fels = hypermat([4 1 size(elementi,'r')]);

Kels(:,:,1) = Greda2D4x4(1, koordinate, elementi, presjekS);
Kels(:,:,2) = Greda2D4x4(2, koordinate, elementi, presjekS);
Kels(:,:,3) = Greda2D4x4(3, koordinate, elementi, presjekS);
Kels(:,:,4) = Greda2D4x4(4, koordinate, elementi, presjekS);

Kels(:,:,5) = Greda2D4x4(5, koordinate, elementi, presjekG);
Kels(:,:,6) = Greda2D4x4(6, koordinate, elementi, presjekG);


Fels(:,:,3) = RavnomjernoOpt(3, koordinate, elementi, 10);
Fels(:,:,4) = RavnomjernoOpt(4, koordinate, elementi, 10);
Fels(:,:,5) = KoncOpt(5, koordinate, elementi, -50, 2);

// ********************* RJESAVANJE ************************

KGlob = zeros(13, 13);
```

```
FGlob = zeros(13, 13);

for i=1:size(elementi, 'r')

  Kel = Kels(:,:,i);

  adresa = SSKEL(i,:);

  for j = 1:4
    for k = 1:4
        if (adresa(j) ~= 0) & (adresa(k) ~= 0) then
          KGlob( adresa(j), adresa(k) ) = ..
          KGlob( adresa(j), adresa(k) ) + Kel(j, k);
        end
    end // for k
  end // for j

//dodavanje vektora opterecenja globalnom vektoru opterecenja
  Fel = Fels(:,:, i);

  for j = 1:4
        if (adresa(j) ~= 0) then
          FGlob( adresa(j) ) = FGlob( adresa(j) ) + Fel(j, 1);
        end
  end // for j
end // for i

// ******* DODAVANJE KONCENTRICNE SILE U CVOROVIMA *********

FGlob( 6 ) = FGlob( 6 ) + 50;

// ************* RJESAVANJE SISTEMA JEDNACINA ****************

// Nema pomjeranja fiksnih oslonaca pa se za rjesavanje
// sistema jednacina uzima "gornji lijevi" blok matrice
// krutosti i odgovarajuci vektor sila

K11 = KGlob(1:8,1:8);
F1 = FGlob(1:8,1);

// rjesenje
U1 = K11 \ F1;

UGlob = zeros(13, 13);
UGlob(1:8, 1) = U1(1:8, 1);

mprintf("\n================== REZULTATI PRORACUNA");
mprintf("\n=== GLOBALNI VEKTOR POMJERANJA CVOROVA");
mprintf("\n\%15.6f", UGlob(1:13,1));

// ************* PRORACUN REAKCIJA ***************************

Reakc = KGlob * UGlob - FGlob;

mprintf("\n============ GLOBALNI VEKTOR REAKCIJA");
mprintf("\n\%11.2f", Reakc(1:13,1));


// **************** POST PROCESSING *************************
// *** PRORACUN POMJERANJA I PRESJECNIH SILA PO ELEMENTIMA ***

for i = 1:size(elementi,'r')
  mprintf('\n============================ STAP %d', i);

  adresa = SSKEL(i,:);

  // uzimanje pomjeranja stapa iz globalnog vektora pomjeranja
  for j = 1:4
      if adresa(j) ~= 0 then
          u( j ) = UGlob( adresa(j) );
      else
          u( j ) = 0;
      end;
  end // for j

  // printanje vektora pomjeranja po stapovima
  mprintf("\nVektor pomjeranja");
  mprintf("\n\%15.6f", u(1:4,1));

  // U opstem slucaju vektor pomjeranja se mora transformisati
  // iz globalnog u lokalni koordinatni sistem, ali ovdje
  // lokalni sistem nije zarotiran pa je
  // u.l = T' * U.g

  // matrica krutosti i vektor vanjskog opterecenja
  Ke = Kels(:,:,i);
  Fe = Fels(:,:,i);

  // proracun vektora sila na stapu
  f = Ke * u - Fe;

  mprintf("\nVektor Sila");
  mprintf("\n\%11.2f", f(1:4,1));

end // for i
```

Listing 5.5: Rezultat proračuna

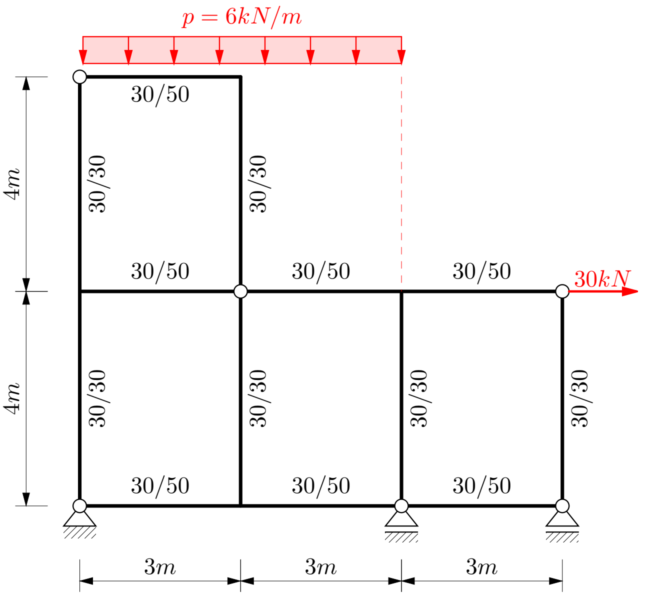
```
================== REZULTATI PRORACUNA
=== GLOBALNI VEKTOR POMJERANJA CVOROVA
      -0.007912
      -0.014966
      -0.003252
      -0.001212
       0.052398
       0.076516
      -0.016352
       0.004651
       0.000000
       0.000000
       0.000000
       0.000000
       0.000000
============ GLOBALNI VEKTOR REAKCIJA
       0.00
       0.00
       0.00
       0.00
      -0.00
      -0.00
       0.00
      -0.00
     -24.77
     -34.70
      74.77
     -85.30
     246.37
============================ STAP 1
Vektor pomjeranja
       0.052398
      -0.014966
       0.000000
       0.000000
Vektor Sila
      85.30
      94.85
     -85.30
     246.37
============================ STAP 2
Vektor pomjeranja
       0.076516
      -0.001212
       0.052398
      -0.014966
Vektor Sila
      -1.34
      90.83
       1.34
     -94.85
============================ STAP 3
Vektor pomjeranja
       0.052398
      -0.007912
       0.000000
      -0.016352
Vektor Sila
      -5.30
      58.78
     -34.70
       0.00
============================ STAP 4
Vektor pomjeranja
       0.076516
      -0.003252
```

```
284         0.052398
285        -0.007912
286  Vektor Sila
287        51.34
288       123.47
289       -81.34
290        75.56
291  =========================== STAP 5
292  Vektor pomjeranja
293         0.000000
294        -0.007912
295         0.000000
296         0.004651
297  Vektor Sila
298        10.94
299      -134.34
300        39.06
301        -0.00
302  =========================== STAP 6
303  Vektor pomjeranja
304         0.000000
305        -0.003252
306         0.000000
307        -0.001212
308  Vektor Sila
309       -35.72
310      -123.47
311        35.72
312       -90.83
```

Pomjeranja Δ_1 i Δ_2 u rezultatu 5.8 predstavljaju relativno pomjeranje čvorova štapa, dok je rezultat na liniji 225 apsolutno pomjeranje čvora što možemo i provjeriti

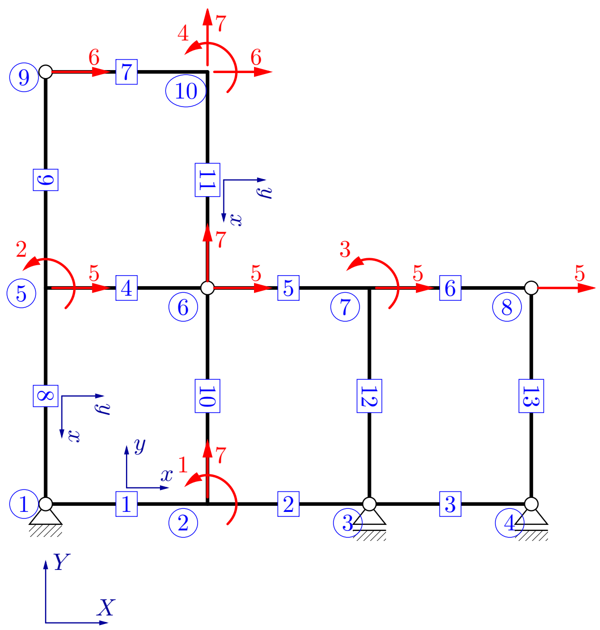
$$\Delta_2 = 0.0241177 \qquad u_6 - u_5 = 0.076516 - 0.052398 = 0.024118$$

Zadatak 6 Metodom deformacija odrediti pomjeranja i presječne sile za konstrukciju na slici 5.23



Slika 5.23

Rješenje



Slika 5.24

a) Rješavanje direktnim asembliranjem matrica krutosti

$$\boxed{1}\quad \begin{array}{cc} & \begin{matrix}0 & 0 & 7 & 1\end{matrix} \\ \begin{matrix}0\\0\\7\\1\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & 0 & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{l^3} & 0 & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} \end{array}$$

$$\boxed{2}\quad \begin{array}{cc} & \begin{matrix}7 & 1 & 0 & 0\end{matrix} \\ \begin{matrix}7\\1\\0\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\boxed{4}\quad \begin{array}{cc} & \begin{matrix}0 & 2 & 7 & 0\end{matrix} \\ \begin{matrix}0\\2\\7\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\boxed{5}\quad \begin{array}{cc} & \begin{matrix}7 & 0 & 0 & 3\end{matrix} \\ \begin{matrix}7\\0\\0\\3\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & 0 & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{l^3} & 0 & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} \end{array}$$

$$\boxed{6}\quad \begin{array}{cc} & \begin{matrix}0 & 3 & 0 & 0\end{matrix} \\ \begin{matrix}0\\3\\0\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\boxed{7}\quad \begin{array}{cc} & \begin{matrix}0 & 0 & 7 & 4\end{matrix} \\ \begin{matrix}0\\0\\7\\4\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & 0 & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{l^3} & 0 & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} \end{array}$$

$$\boxed{8}\quad \begin{array}{cc} & \begin{matrix}5 & 2 & 0 & 0\end{matrix} \\ \begin{matrix}5\\2\\0\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\boxed{9}\quad \begin{array}{cc} & \begin{matrix}6 & 0 & 5 & 2\end{matrix} \\ \begin{matrix}6\\0\\5\\2\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & 0 & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{l^3} & 0 & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} \end{array}$$

$$\boxed{10}\quad \begin{array}{cc} & \begin{matrix}5 & 0 & 0 & 1\end{matrix} \\ \begin{matrix}5\\0\\0\\1\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & 0 & -\frac{3EI}{l^3} & \frac{3EI}{l^2} \\ 0 & 0 & 0 & 0 \\ -\frac{3EI}{l^3} & 0 & \frac{3EI}{l^3} & -\frac{3EI}{l^2} \\ \frac{3EI}{l^2} & 0 & -\frac{3EI}{l^2} & \frac{3EI}{l} \end{bmatrix} \end{array}$$

$$\boxed{11}\quad \begin{array}{cc} & \begin{matrix}6 & 4 & 5 & 0\end{matrix} \\ \begin{matrix}6\\4\\5\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\boxed{12}\quad \begin{array}{cc} & \begin{matrix}5 & 3 & 0 & 0\end{matrix} \\ \begin{matrix}5\\3\\0\\0\end{matrix} & \begin{bmatrix} \frac{3EI}{l^3} & \frac{3EI}{l^2} & -\frac{3EI}{l^3} & 0 \\ \frac{3EI}{l^2} & \frac{3EI}{l} & -\frac{3EI}{l^2} & 0 \\ -\frac{3EI}{l^3} & -\frac{3EI}{l^2} & \frac{3EI}{l^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$3EI_G/l_G = 3 \cdot 3 \cdot 10^7 \cdot 0.003125 = 93750$$
$$3EI_G/l_G^2 = 3 \cdot 3 \cdot 10^7 \cdot 0.003125^2 = 31250$$
$$3EI_G/l_G^3 = 3 \cdot 3 \cdot 10^7 \cdot 0.003125^3 = 10416.67$$
$$3EI_S/l_S = 3 \cdot 3 \cdot 10^7 \cdot 0.000675 = 15187.5$$
$$3EI_S/l_S^2 = 3 \cdot 3 \cdot 10^7 \cdot 0.000675^2 = 3796.88$$
$$3EI_S/l_S^3 = 3 \cdot 3 \cdot 10^7 \cdot 0.000675^3 = 949.22$$

$$
\begin{array}{ccccccc}
\varphi_2 & \varphi_5 & \varphi_7 & \varphi_{10} & u_1 & u_2 & u_3
\end{array}
$$

$$
\begin{bmatrix}
202687.50 & 0 & 0 & 0 & 3796.88 & 0 & 0 \\
 & 124125.00 & 0 & 0 & 0 & 3796.88 & -31250.00 \\
 & & 202687.50 & 0 & 3796.88 & 0 & 31250.00 \\
 & & & 108937.50 & -3796.88 & 3796.88 & -31250.00 \\
 & & & & 4746.1 & -1898.44 & 0 \\
 & \text{simetricno} & & & & 1898.44 & 0 \\
 & & & & & & 52083.34
\end{bmatrix}
\begin{bmatrix} \varphi_2 \\ \varphi_5 \\ \varphi_7 \\ \varphi_{10} \\ u_1 \\ u_2 \\ u_3 \end{bmatrix}
=
\begin{bmatrix} 0 \\ 0 \\ 6.75 \\ 6.75 \\ 30.00 \\ 0 \\ -18.00 \end{bmatrix}
$$

Slika 5.25: Sistem jednačina

$$K_{11} = \frac{3EI_G}{l_G} + \frac{3EI_G}{l_G} + \frac{3EI_S}{l_S} = 202687.5$$
$$K_{12} = 0$$
$$K_{13} = 0$$
$$K_{14} = 0$$
$$K_{15} = \frac{3EI_S}{l_S^2} = 3796.88$$
$$K_{16} = 0$$
$$K_{17} = -\frac{3EI_G}{l_G^2} + \frac{3EI_G}{l_G^2} = 0$$

$$K_{22} = \frac{3EI_G}{l_G} + \frac{3EI_S}{l_S} + \frac{3EI_S}{l_S} = 124125$$
$$K_{23} = 0$$
$$K_{24} = 0$$
$$K_{25} = \frac{3EI_S}{l_S^2} - \frac{3EI_S}{l_S^2} = 0$$
$$K_{26} = \frac{3EI_S}{l_S^2} = 3796.88$$
$$K_{27} = -\frac{3EI_G}{l_G^2} = -31250$$

$$K_{33} = \frac{3EI_G}{l_G} + \frac{3EI_G}{l_G} + \frac{3EI_S}{l_S} = 202687.5$$
$$K_{34} = 0$$
$$K_{35} = \frac{3EI_S}{l_S^2} = 3796.88$$
$$K_{36} = 0$$
$$K_{37} = \frac{3EI_G}{l_G^2} = 31250$$

$$K_{44} = \frac{3EI_G}{l_G} + \frac{3EI_S}{l_S} = 108937.5$$
$$K_{45} = -\frac{3EI_S}{l_S^2} = -3796.88$$
$$K_{46} = \frac{3EI_S}{l_S^2} = 3796.88$$
$$K_{47} = -\frac{3EI_G}{l_G^2} = -31250$$

$$K_{55} = \frac{3EI_S}{l_S^3} + \frac{3EI_S}{l_S^3} + \frac{3EI_S}{l_S^3} + \frac{3EI_S}{l_S^3} + \frac{3EI_S}{l_S^3} = 4746.1$$
$$K_{56} = -\frac{3EI_S}{l_S^3} - \frac{3EI_S}{l_S^3} = -1898.44$$
$$K_{57} = 0$$

$$K_{66} = \frac{3EI_S}{l_S^3} + \frac{3EI_S}{l_S^3} = 1898.44$$
$$K_{67} = 0$$

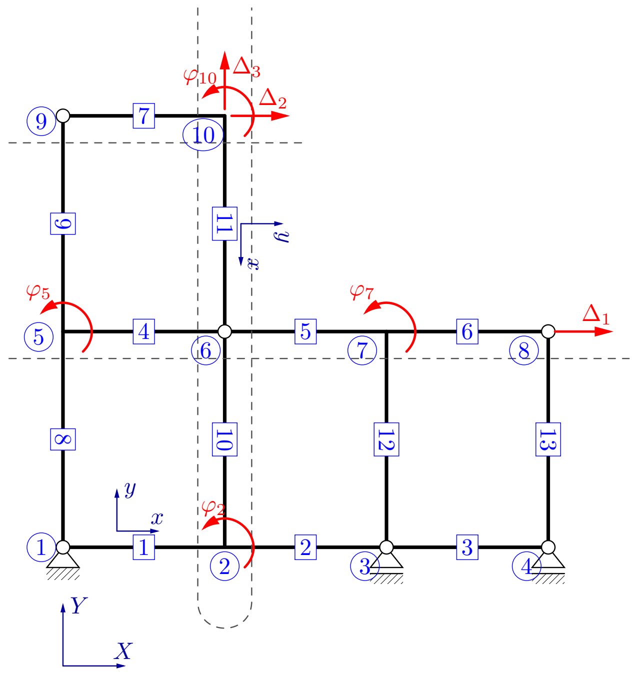
$$K_{77} = \frac{3EI_G}{l_G^3} + \frac{3EI_G}{l_G^3} + \frac{3EI_G}{l_G^3} + \frac{3EI_G}{l_G^3} + \frac{3EI_G}{l_G^3} = 52083.35$$

Rješenje sistema jednačina

$$
\boldsymbol{u} = \begin{bmatrix} -0.0002197 \\ -0.0006170 \\ -0.0000602 \\ -0.0002322 \\ 0.0117308 \\ 0.0134291 \\ -0.0008190 \end{bmatrix}
$$

Sad ćemo riješiti ovaj sistem postavljanjem Takabejevih jednačina i uporediti rezultate, a nakon toga ćemo odrediti presječne sile po štapovima.

b) Rješavanje postavljanjem Takabejevih jednačina

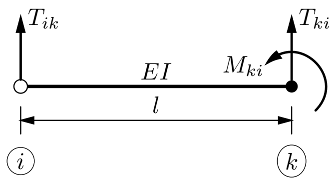


Slika 5.26

Pomjeranja koja se traže

- Rotacije čvorova $\varphi_2, \varphi_5, \varphi_7, \varphi_{10}$
- Translatorna pomjeranja $\Delta_1, \Delta_2, \Delta_3$

Takabejeve jednačine



Slika 5.27

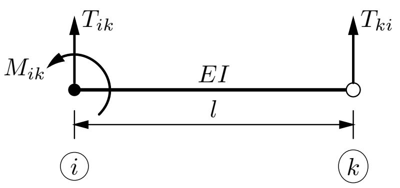
$$M_{ik} = 0$$

$$M_{ki} = 1.5k_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - M_{ki}^{\circ}$$

$$T_{ik} = 0.5\bar{k}_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$

$$T_{ki} = -0.5\bar{k}_{ik}\left(\varphi_k - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

gdje je $k_{ik} = \dfrac{2EI}{l_{ik}}, \bar{k}_{ik} = \dfrac{6EI}{l_{ik}^2}, \bar{k}_{ik} = \dfrac{3k_{ik}}{l_{ik}}$



Slika 5.28

$$M_{ik} = 1.5k_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - M_{ik}^{\circ}$$

$$M_{ki} = 0$$

$$T_{ik} = 0.5\bar{k}_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ik}^{\circ}$$

$$T_{ki} = -0.5\bar{k}_{ik}\left(\varphi_i - \frac{\Delta_{ik}}{l_{ik}}\right) - T_{ki}^{\circ}$$

Postavljanje jednačina ravnoteže

Rotacija čvora 2

$$M_{21} = 93750.0{\cdot}\varphi_2 - 31250 \quad \cdot \Delta_3$$
$$M_{26} = 15187.5{\cdot}\varphi_2 - 3796.875 \cdot \Delta_1$$
$$M_{23} = 93750.0{\cdot}\varphi_2 - 31250 \quad \cdot (-\Delta_3)$$

$$\sum_{k=1,3,6} M_{2k} = 202687.5 \cdot \varphi_2 - 3796.875 \cdot \Delta_1 = 0$$

Rotacija čvora 5

$$M_{51} = 15187.5{\cdot}\varphi_5 - 3796.875 \cdot \Delta_1$$
$$M_{56} = 93750.0{\cdot}\varphi_5 - 31250 \quad \cdot \Delta_3$$
$$M_{59} = 15187.5{\cdot}\varphi_5 - 3796.875 \cdot \Delta_2$$

$$\sum_{k=1,6,9} M_{5k} = 0$$

$$124125 \cdot \varphi_5 - 3796.875 \cdot \Delta_1 - 3796.875 \cdot \Delta_2 - 31250 \cdot \Delta_3 = 0$$

Rotacija čvora 7

$$M_{76} = 93750.0{\cdot}\varphi_7 - 31250 \quad \cdot (-\Delta_3) + 6.75$$
$$M_{73} = 15187.5{\cdot}\varphi_7 - 3796.875 \cdot \Delta_1$$
$$M_{78} = 93750.0{\cdot}\varphi_7$$

$$\sum_{k=6,3,8} M_{7k} = 0$$

$$202687.5 \cdot \varphi_7 - 3796.875 \cdot \Delta_1 + 31250 \cdot \Delta_3 = -6.75$$

Rotacija čvora 10

$$M_{10-9} = 93750.0{\cdot}\varphi_{10} - 31250 \quad \cdot (-\Delta_3) + 6.75$$
$$M_{10-6} = 15187.5{\cdot}\varphi_{10} - 3796.875 \cdot \Delta_2$$

$$\sum_{k=9,6} M_{10-k} = 0$$

$$108937.5 \cdot \varphi_{10} - 3796.875 \cdot \Delta_2 + 31250 \cdot \Delta_3 = -6.75$$

Relativno pomjeranje Δ_1

$$T_{51} = -3796.875{\cdot}\varphi_5 + 949.219 \cdot \Delta_1$$
$$T_{62} = -3796.875{\cdot}\varphi_2 + 949.219 \cdot \Delta_1$$
$$T_{73} = -3796.875{\cdot}\varphi_7 + 949.219 \cdot \Delta_1$$
$$T_{84} = 0$$

$$\sum_{i,k} T_{ik} = 0$$

$$2847.657{\cdot}\Delta_1 - 3796.875{\cdot}\varphi_2 - 3796.875{\cdot}\varphi_5 - 3796.875{\cdot}\varphi_7 = 30$$

Relativno pomjeranje Δ_2

$$T_{95} = -3796.875{\cdot}\varphi_5 + 949.219 \quad \cdot \Delta_2$$
$$T_{10-6} = -3796.875{\cdot}\varphi_{10} + 949.219 \cdot \Delta_2$$

$$
\begin{array}{c}
\begin{array}{ccccccc}
\varphi_2 & \varphi_5 & \varphi_7 & \varphi_{10} & \Delta_1 & \Delta_2 & \Delta_3
\end{array}\\
\begin{bmatrix}
202687.50 & 0 & 0 & 0 & -3796.88 & 0 & 0\\
0 & 124125.00 & 0 & 0 & -3796.88 & -3796.88 & -31250.00\\
0 & 0 & 202687.50 & 0 & -3796.88 & 0 & 31250.00\\
0 & 0 & 0 & 108937.50 & 0 & -3796.88 & -31250.00\\
-3796.88 & -3796.88 & -3796.88 & 0 & 2847.66 & 0 & 0\\
0 & -3796.88 & 0 & -3796.88 & 0 & 1898.44 & 0\\
0 & -31250.00 & 31250.00 & -31250.00 & 0 & 0 & 52083.34
\end{bmatrix}
\end{array}
\begin{bmatrix}\varphi_2\\ \varphi_5\\ \varphi_7\\ \varphi_{10}\\ \Delta_1\\ \Delta_2\\ \Delta_3\end{bmatrix}
=
\begin{bmatrix}0\\ 0\\ -6.75\\ -6.75\\ 30.00\\ 0\\ 18.00\end{bmatrix}
$$

$$\sum_{i,k} T_{ik} = 0$$

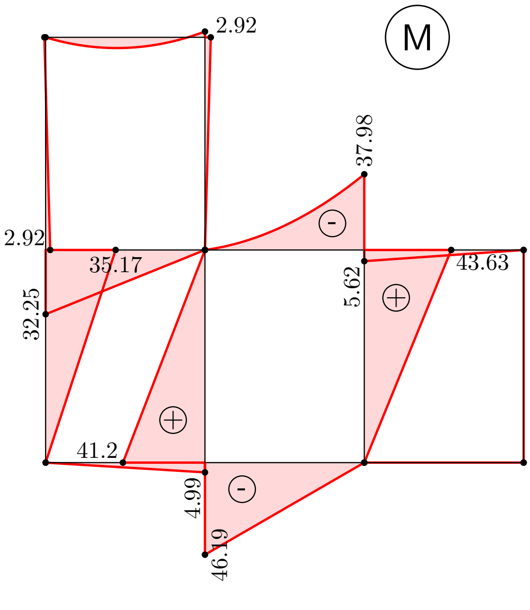
$$1898.438 \cdot \Delta_2 - 3796.875 \cdot \varphi_5 - 3796.875 \cdot \varphi_{10} = 0$$

Relativno pomjeranje Δ_3

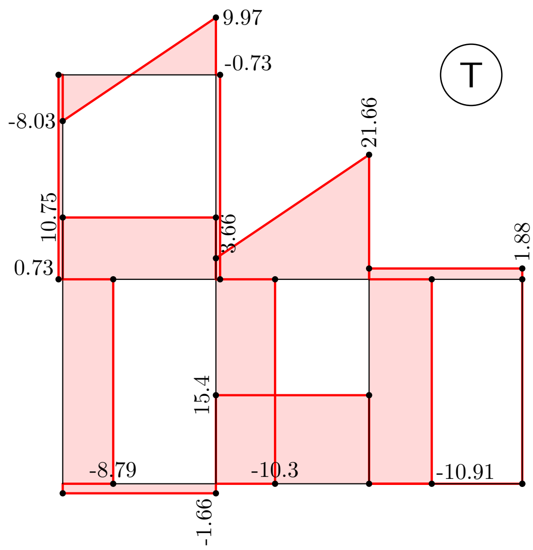
$$
\begin{aligned}
T_{10-9} &= -31250{\cdot}\varphi_{10} + 10416.667 \cdot \Delta_3 - 11.25\\
T_{65} &= -31250{\cdot}\varphi_5 + 10416.667 \cdot \Delta_3\\
T_{21} &= -31250{\cdot}\varphi_2 + 10416.667 \cdot \Delta_3\\
T_{23} &= 31250{\cdot}\ \varphi_2 - 10416.667 \cdot (-\Delta_3)\\
T_{67} &= 31250{\cdot}\ \varphi_7 - 10416.667 \cdot (-\Delta_3) - 6.75
\end{aligned}
$$

$$\sum_{i,k} T_{ik} = 0$$

$$52083.335 \cdot \Delta_3 - 31250 \cdot \varphi_5 + 31250 \cdot \varphi_7 - 31250 \cdot \varphi_{10} = 18$$



Slika 5.29: Dijagram momenata



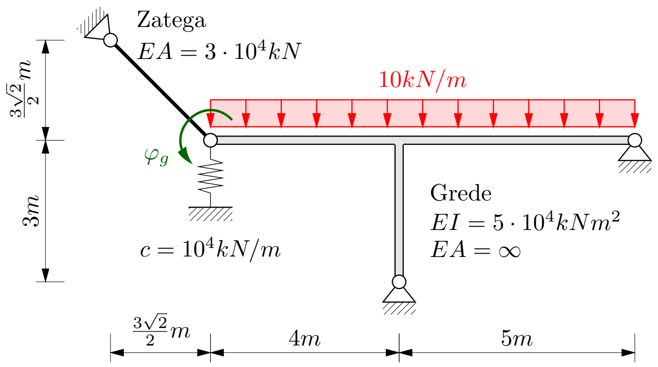
Slika 5.30: Dijagram poprečnih sila

Zadatak 7 Konstrukciju na slici 5.31 proračunati koristeći metodu deformacija. Za gredne elemente pretpostaviti da su aksijalno kruti.

a) Odrediti moguće stepene slobode kretanja,

b) Proračunati pomjeranja u prethodno definisanim stepenima slobode kretanja. Proračunati takođe i rotaciju kraja grede φ_g.

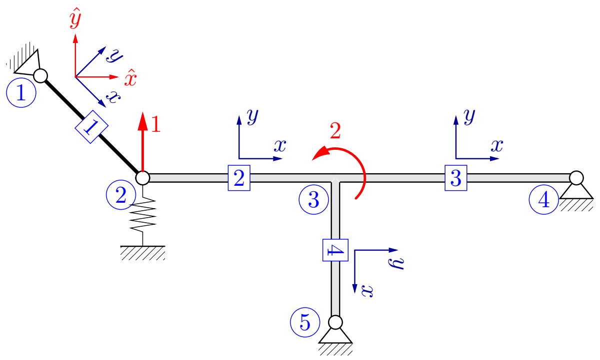
c) Proračunati presječne sile na krajevima elemenata i nacrtati dijagrame presječnih sila.



Slika 5.31

Rješenje

a) Stepeni slobode kretanja su označeni na slici 5.32



Slika 5.32

b) Proračun pomjeranja

1

$$\begin{array}{c|cccc} & 0 & 0 & 0 & 1 \\ \hline 0 & c^2 & cs & -c^2 & -cs \\ 0 & cs & s^2 & -cs & -s^2 \\ 0 & -c^2 & -cs & c^2 & cs \\ 1 & -cs & -s^2 & cs & s^2 \end{array} \quad \frac{EA}{l_1}$$

2

$$\begin{array}{c|cccc} & 1 & 0 & 0 & 2 \\ \hline 1 & \frac{3EI}{l_2^3} & 0 & -\frac{3EI}{l_2^3} & \frac{3EI}{l_2^2} \\ 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{3EI}{l_2^3} & 0 & \frac{3EI}{l_2^3} & -\frac{3EI}{l_2^2} \\ 2 & \frac{3EI}{l_2^2} & 0 & -\frac{3EI}{l_2^2} & \frac{3EI}{l_2} \end{array}$$

3

$$\begin{array}{c|cccc} & 0 & 2 & 0 & 0 \\ \hline 0 & \frac{3EI}{l_3^3} & \frac{3EI}{l_3^2} & -\frac{3EI}{l_3^3} & 0 \\ 2 & \frac{3EI}{l_3^2} & \frac{3EI}{l_3} & -\frac{3EI}{l_3^2} & 0 \\ 0 & -\frac{3EI}{l_3^3} & -\frac{3EI}{l_3^2} & \frac{3EI}{l_3^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

4

$$\begin{array}{c|cccc} & 0 & 2 & 0 & 0 \\ \hline 0 & \frac{3EI}{l_4^3} & \frac{3EI}{l_4^2} & -\frac{3EI}{l_4^3} & 0 \\ 2 & \frac{3EI}{l_4^2} & \frac{3EI}{l_4} & -\frac{3EI}{l_4^2} & 0 \\ 0 & -\frac{3EI}{l_4^3} & -\frac{3EI}{l_4^2} & \frac{3EI}{l_4^3} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}$$

$$K_{11} = \frac{EA}{l_1}s^2 + \frac{3EI}{l_2^3} + 10^4 = 10^4 \cdot \frac{1}{2} + \frac{15}{64} \cdot 10^4 + 10^4 = \frac{111}{64} \cdot 10^4$$

$$K_{12} = \frac{3EI}{l_2^2} = \frac{15}{16} \cdot 10^4$$

$$K_{22} = \frac{3EI}{l_2} + \frac{3EI}{l_3} + \frac{3EI}{l_4} = \frac{47}{4} \cdot 10^4$$

pa dobijamo

$$\boldsymbol{K} = 10^4 \cdot \begin{bmatrix} 1.7344 & 0.9375 \\ 0.9375 & 11.75 \end{bmatrix}$$

Globalni vektor sila

Štap 2

Za kruto vezan štap

$$M_2^\bullet = -\frac{q \cdot l^2}{12} = -\frac{10 \cdot 4^2}{12} = -13.333$$

$$M_3^\bullet = \frac{q \cdot l^2}{12} = \frac{10 \cdot 4^2}{12} = 13.333$$

$$V_2^\bullet = -\frac{q \cdot l}{2} = -\frac{10 \cdot 4}{2} = -20.0$$

$$V_3^\bullet = -\frac{q \cdot l}{2} = -\frac{10 \cdot 4}{2} = -20.0$$

Za štap 2 zglobno vezan u čvoru 2

$$M_2^\circ = 0$$

$$M_3^\circ = M_3^\bullet - 0.5 \cdot M_2^\bullet = 13.333 - 0.5 \cdot (-13.333) = 20.0$$

$$V_2^\circ = V_2^\bullet - 1.5\frac{M_2^\bullet}{l_2} = -20.0 - 1.5\frac{-13.333}{4} = -15.0$$

$$V_3^\circ = V_3^\bullet + 1.5\frac{M_2^\bullet}{l_2} = -20.0 + 1.5\frac{-13.333}{4} = -25.0$$

Vektor sila štapa 2 za globalni sistem jednačina, formiran na osnovu globalnih stepeni slobode kretanja

$$\boldsymbol{f}^2 = \begin{bmatrix} -15.0 \\ 20.0 \end{bmatrix}$$

Štap 3

Za kruto vezan štap

$$M_3^\bullet = -\frac{q \cdot l^2}{12} = -\frac{10 \cdot 5^2}{12} = -20.8333$$

$$M_4^\bullet = \frac{q \cdot l^2}{12} = \frac{10 \cdot 5^2}{12} = 20.8333$$

$$V_3^\bullet = -\frac{q \cdot l}{2} = -\frac{10 \cdot 5}{2} = -25.0$$

$$V_4^\bullet = -\frac{q \cdot l}{2} = -\frac{10 \cdot 5}{2} = -25.0$$

Za štap 3 zglobno vezan u čvoru 4

$$M_3^\circ = M_3^\bullet - 0.5 \cdot M_4^\bullet = -20.8333 - 0.5 \cdot (20.8333) = -31.25$$

$$M_4^\circ = 0$$

$$V_3^\circ = V_3^\bullet - 1.5\frac{M_4^\bullet}{l_3} = -25.0 - 1.5\frac{20.8333}{5} = -31.25$$

$$V_4^\circ = V_4^\bullet + 1.5\frac{M_4^\bullet}{l_3} = -25.0 + 1.5\frac{20.8333}{5} = -18.75$$

Vektor sila štapa 3 za globalni sistem jednačina, formiran na osnovu globalnih stepeni slobode kretanja

$$\boldsymbol{f}^3 = \begin{bmatrix} 0 \\ -31.25 \end{bmatrix}$$

Konačno dobijamo globalni vektor sila

$$\boldsymbol{f} = \begin{bmatrix} -15.0 \\ 20.0 \end{bmatrix} + \begin{bmatrix} 0 \\ -31.25 \end{bmatrix} = \begin{bmatrix} -15 \\ -11.25 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{Ku} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} -0.000850 \\ -0.000028 \end{bmatrix}$$

Rotacija grede 2 u čvoru 2

$$\varphi_2 = -0.5\varphi_3 + 1.5\frac{\Delta_{23}}{l_2} + 0.25\frac{M^{\bullet}_{23}}{k_{23}}$$

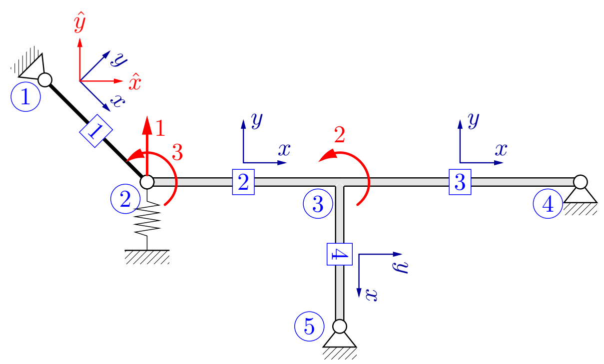
$$\Delta_{23} = v_k - v_i = 0 - (-0.000850) = 0.000850$$

$$k_{23} = \frac{EI}{l_2} = \frac{5 \cdot 10^4}{4} = 1.25 \cdot 10^4$$

$$\varphi_2 = -0.5 \cdot (-0.000028) + 1.5\frac{0.000850}{4} + 0.25\frac{-13.333}{1.25 \cdot 10^4}$$
$$= 0.000066$$

Uvođenje stepena slobode kretanja 3

Druga mogućnost za proračun rotacije grede 2 u čvoru 2 je uvođenje stepena slobode kretanja 3, koje ulazi u vektor nepoznatih pomjeranja, kako je prikazano na slici 5.33 U ovom slučaju za štap 2 koristimo matricu krutosti štapa kruto vezanog na oba kraja. Primijetimo da iako postoji SSK 3 u čvoru 2 štap 1 nema odgovarajući SSK i ne doprinosti rotacionoj krutosti (SSK 3) u čvoru 2



Slika 5.33

$$\begin{array}{c|c} & \begin{matrix} 0 & 0 & 0 & 1 \end{matrix} \\ \boxed{1} \; \begin{matrix} 0 \\ 0 \\ 0 \\ 1 \end{matrix} & \begin{bmatrix} c^2 & cs & -c^2 & -cs \\ cs & s^2 & -cs & -s^2 \\ -c^2 & -cs & c^2 & cs \\ -cs & -s^2 & cs & s^2 \end{bmatrix} \frac{EA}{l_1} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 1 & 3 & 0 & 2 \end{matrix} \\ \boxed{2} \; \begin{matrix} 1 \\ 3 \\ 0 \\ 2 \end{matrix} & \begin{bmatrix} \frac{12EI}{l_2^3} & \frac{6EI}{l_2^2} & -\frac{12EI}{l_2^3} & \frac{6EI}{l_2^2} \\ \frac{6EI}{l_2^2} & \frac{4EI}{l_2} & -\frac{6EI}{l_2^2} & \frac{2EI}{l_2} \\ -\frac{12EI}{l_2^3} & -\frac{6EI}{l_2^2} & \frac{12EI}{l_2^3} & -\frac{6EI}{l_2^2} \\ \frac{6EI}{l_2^2} & \frac{2EI}{l_2} & -\frac{6EI}{l_2^2} & \frac{4EI}{l_2} \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 0 & 2 & 0 & 0 \end{matrix} \\ \boxed{3} \; \begin{matrix} 0 \\ 2 \\ 0 \\ 0 \end{matrix} & \begin{bmatrix} \frac{3EI}{l_3^3} & \frac{3EI}{l_3^2} & -\frac{3EI}{l_3^3} & 0 \\ \frac{3EI}{l_3^2} & \frac{3EI}{l_3} & -\frac{3EI}{l_3^2} & 0 \\ -\frac{3EI}{l_3^3} & -\frac{3EI}{l_3^2} & \frac{3EI}{l_3^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$\begin{array}{c|c} & \begin{matrix} 0 & 2 & 0 & 0 \end{matrix} \\ \boxed{4} \; \begin{matrix} 0 \\ 2 \\ 0 \\ 0 \end{matrix} & \begin{bmatrix} \frac{3EI}{l_4^3} & \frac{3EI}{l_4^2} & -\frac{3EI}{l_4^3} & 0 \\ \frac{3EI}{l_4^2} & \frac{3EI}{l_4} & -\frac{3EI}{l_4^2} & 0 \\ -\frac{3EI}{l_4^3} & -\frac{3EI}{l_4^2} & \frac{3EI}{l_4^3} & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

$$K_{11} = \frac{EA}{l_1}s^2 + \frac{12EI}{l_2^3} + 10^4 =$$
$$10^4 \cdot \frac{1}{2} + \frac{60}{64} \cdot 10^4 + 10^4 = \frac{156}{64} \cdot 10^4$$

$$K_{12} = \frac{6EI}{l_2^2} = \frac{30}{16} \cdot 10^4$$

$$K_{13} = \frac{6EI}{l_2^2} = \frac{30}{16} \cdot 10^4$$

$$K_{22} = \frac{4EI}{l_2} + \frac{3EI}{l_3} + \frac{3EI}{l_4} = 13 \cdot 10^4$$

$$K_{23} = \frac{2EI}{l_2} = \frac{5}{2}10^4$$

$$K_{33} = \frac{4EI}{l_2} = 5 \cdot 10^4$$

pa dobijamo

$$\boldsymbol{K} = 10^4 \cdot \begin{bmatrix} 2.4375 & 1.875 & 1.875 \\ & 13.000 & 2.500 \\ \text{simetricno} & & 5.000 \end{bmatrix}$$

Globalni vektor sila

$$\boldsymbol{f} = \begin{bmatrix} -20.000 \\ 13.333 \\ -13.333 \end{bmatrix} + \begin{bmatrix} 0 \\ -31.25 \\ 0 \end{bmatrix} = \begin{bmatrix} -20.000 \\ -17.917 \\ -13.333 \end{bmatrix}$$

Rješenjem sistema $\boldsymbol{Ku} = \boldsymbol{f}$ dobijaju se pomjeranja

$$\boldsymbol{u} = \begin{bmatrix} -0.000850 \\ -0.000028 \\ 0.000066 \end{bmatrix}$$

odakle možemo zaključiti da je $\boldsymbol{u}_3 = \varphi_2$ koje je proračunato u prethodnoj sekciji.

c) Presječne sile na krajevima elemenata

Štap 1

Transformacija pomjeranja čvora $\hat{\boldsymbol{u}}_n$ iz globalnog koordinatnog sistema u lokalni koordinatni sistem (vektor pomjeranja $\boldsymbol{u}^e_n$).

$$\hat{\boldsymbol{u}}_n = \begin{bmatrix} c & -s \\ s & c \end{bmatrix} \boldsymbol{u}^e_n \quad \Rightarrow \quad \boldsymbol{u}^e_n = \begin{bmatrix} c & s \\ -s & c \end{bmatrix} \hat{\boldsymbol{u}}_n$$

gdje je

$$\hat{\boldsymbol{u}}_n = \begin{bmatrix} \hat{u}_{nx} \\ \hat{u}_{ny} \end{bmatrix}, \qquad \boldsymbol{u}^e_n = \begin{bmatrix} u^e_{nx} \\ u^e_{ny} \end{bmatrix}$$

$(\alpha = -45°)$

Pomjeranje čvora 2 u lokalnom koordinatnom sistemu štapa 1

$$\boldsymbol{u}_2^1 = \begin{bmatrix} u_{2x}^1 \\ u_{2y}^1 \end{bmatrix} = \frac{\sqrt{2}}{2} \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -0.000850 \end{bmatrix} = \begin{bmatrix} 0.000601 \\ -0.000601 \end{bmatrix}$$

Vektor pomjeranja čvorova štapa 1 u lokalnom koordinatnom sistemu

$$\boldsymbol{u}^1 = \begin{bmatrix} u_{1x}^1 \\ u_{2x}^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.000601 \end{bmatrix}$$

$$\boldsymbol{f}^1 = \boldsymbol{K}^1 \boldsymbol{u}^1 = 10^4 \cdot \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.000601 \end{bmatrix} = \begin{bmatrix} -6.01 \\ 6.01 \end{bmatrix}$$

Štap 2

$$\boldsymbol{f}^2 = \boldsymbol{K}^2 \boldsymbol{u}^2 - \boldsymbol{s}_2^{\bullet}$$

$$= \frac{5 \cdot 10^4}{64} \cdot \begin{bmatrix} 12 & 24 & -12 & 24 \\ 24 & 64 & -24 & 32 \\ -12 & -24 & 12 & -24 \\ 24 & 32 & -24 & 64 \end{bmatrix} \begin{bmatrix} -0.000850 \\ 0.000066 \\ 0 \\ -0.000028 \end{bmatrix} - \begin{bmatrix} -20.000 \\ -13.333 \\ -20.000 \\ 13.333 \end{bmatrix}$$

$$= \begin{bmatrix} 12.74 \\ 0 \\ 27.26 \\ -29.02 \end{bmatrix}$$

Sad ćemo za vježbu proračunati vektor sila štapa 2 koristeći rezultat dobijen u prvoj sekciji kad smo štap 2 posmatrali kao zglobno vezan u čvoru 2, dakle rotacija φ_2 je izbačena statičkom kondenzacijom i vektor pomjeranja štapa je

$$\begin{bmatrix} v_2 \\ v_3 \\ \varphi_3 \end{bmatrix}$$

$$\boldsymbol{f}^2 = \boldsymbol{K}^2 \boldsymbol{u}^2 - \boldsymbol{s}_2^{\circ}$$

$$= \frac{5 \cdot 10^4}{64} \cdot \begin{bmatrix} 3 & -3 & 12 \\ -3 & 3 & -12 \\ 12 & -12 & 48 \end{bmatrix} \begin{bmatrix} -0.000850 \\ 0 \\ -0.000028 \end{bmatrix} - \begin{bmatrix} -15.0 \\ -25.0 \\ 20.0 \end{bmatrix}$$

$$= \begin{bmatrix} 12.74 \\ 27.25 \\ -29.02 \end{bmatrix}$$

Štap 3

$$\boldsymbol{f}^3 = \boldsymbol{K}^3 \boldsymbol{u}^3 - \boldsymbol{s}_3^{\circ}$$

$$= \frac{3 \cdot 5 \cdot 10^4}{125} \cdot \begin{bmatrix} 1 & 5 & -1 & 0 \\ 5 & 25 & -5 & 0 \\ -1 & -5 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.000028 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} -31.25 \\ -31.25 \\ -18.751 \\ 0 \end{bmatrix}$$

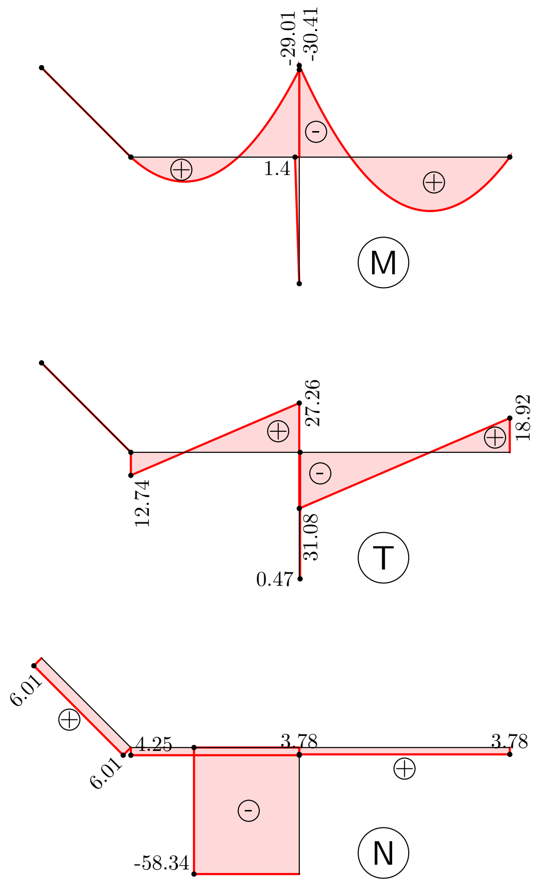
$$= \begin{bmatrix} 31.08 \\ 30.41 \\ 18.92 \\ 0 \end{bmatrix}$$

Štap 4

$$\boldsymbol{f}^4 = \boldsymbol{K}^4 \boldsymbol{u}^4 - \boldsymbol{s}_4^{\circ}$$

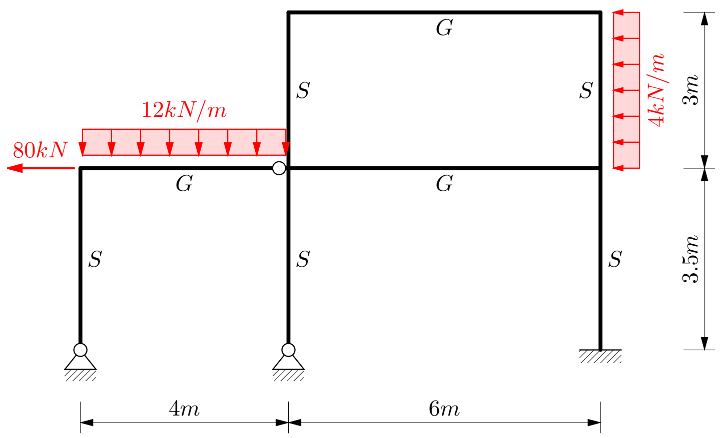
$$= \frac{3 \cdot 5 \cdot 10^4}{27} \cdot \begin{bmatrix} 1 & 3 & -1 & 0 \\ 3 & 9 & -3 & 0 \\ -1 & -3 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -0.000028 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -0.467 \\ -1.400 \\ 0.467 \\ 0 \end{bmatrix}$$



Slika 5.34

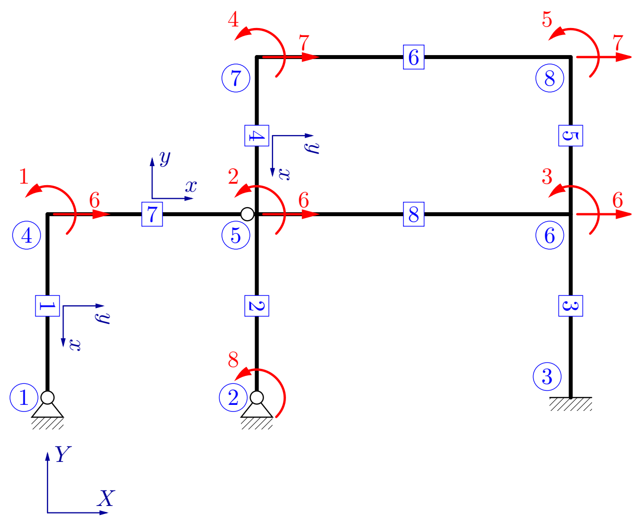
Zadatak 8 Proračunati pomjeranja, reakcije i presječne sile na konstrukciji sa slike 5.35 primjenom metode deformacija. Pretpostaviti da su svi štapovi aksijalno kruti.



$E = 3 \cdot 10^7 kN/m^2$
$G : 30 \times 50cm$
$S : 40 \times 40cm$

Slika 5.35

Rješenje Stepeni slobode kretanja su dati na slici 5.36. Sa pretpostavkom o aksijalnoj krutosti štapova čvorovi 4, 5 i 6 imaju isti stepen slobode kretanja u X pravcu 6, a čvorovi 7 i 8 imaju isti SSK 7. Pošto su i stubovi aksijalno kruti nijedan čvor nema mogućnost pomjeranja u pravcu Y.



Slika 5.36

Primjetimo da su štapovi 1 i 2 na isti način vezani, međutim u svrhu vježbanja, za štap 1 ćemo koristiti matricu krutosti elementa kojem je oslobađanje momenta na kraju štapa uzeto u obzir statičkom kondenzacijom sistema jednačina nakon čega rotacioni SSK kod čvora j (na štapu sa čvorovima $i - j$) ne figurira u sistemu jednačina. (Isto i za štap 7). Za štap 2 ćemo koristiti matricu krutosti elementa kruto vezanog na oba kraja pa ćemo zato kod čvora 2 imati SSK 8 koji ulazi u vektor nepoznatih pomjeranja $\boldsymbol{u}_n$, vidjeti jednačinu 5.9 Za štap 3 ćemo koristiti istu matricu krutosti kao za štap 2 međutim pošto je štap 3 uklješten kod čvora 3,

$$\begin{bmatrix} \boldsymbol{K}_{nn} & \boldsymbol{K}_{np} \\ \boldsymbol{K}_{pn} & \boldsymbol{K}_{pp} \end{bmatrix} \begin{bmatrix} \boldsymbol{u}_n \\ \boldsymbol{u}_p \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_p \\ \boldsymbol{F}_n \end{bmatrix} \tag{5.9}$$

$\boldsymbol{u}_n$ - vektor nepoznatih pomjeranja
$\boldsymbol{u}_p$ - vektor poznatih pomjeranja u osloncima
$\boldsymbol{F}_p$ - vektor poznatih sila
$\boldsymbol{F}_n$ - vektor nepoznatih sila - reakcije

njegovo rotaciono pomjeranje kod čvora 3 ulazi u vektor poznatih pomjeranja $\boldsymbol{u}_p$, a kako su sva poznata pomjeranja $\boldsymbol{u}_p = \boldsymbol{0}$ formiramo samo matricu $\boldsymbol{K}_{nn}$ koja odgovara vektoru nepoznatih pomjeranja tako da nismo ni obilježili stepene slobode kretanja koji odgovaraju poznatim pomjeranjima.

Iz globalnog sistema jednačina 5.9, uvrštavajući poznata pomjeranja dobijamo redukovani sistem

$$\boldsymbol{K}_{nn} \cdot \boldsymbol{u}_n + \boldsymbol{K}_{np} \cdot \boldsymbol{0} = \boldsymbol{F}_p \Rightarrow \boldsymbol{u}_n = \boldsymbol{K}_{nn}^{-1} \boldsymbol{F}_p$$

Listing 5.6: Matrica krutosti štapa sa otpuštenim momentom u čvoru k, `Greda2DTiMi_Tk.sci`

```
function rezultat = Greda2DTiMi_Tk(idx, koordinate, elementi,
        presjek)
// Greda2DTiMi_Tk - Matrica krutosti ravnog grednog
// aksijalno krutog stapa
// cvorovi i-k ; u cvoru k otpusten momenat

Em = presjek(1);
I = presjek(2);

x1 = koordinate(elementi(idx, 1), 1);
y1 = koordinate(elementi(idx, 1), 2);
x2 = koordinate(elementi(idx, 2), 1);
y2 = koordinate(elementi(idx, 2), 2);

duzina = sqrt((x2 - x1)^2 + (y2 - y1)^2);

km3   = 3 * Em * I / duzina^3;
km3L2 = km3 * duzina;
km3L  = km3L2 * duzina;

rezultat = [
 km3    km3L2  -km3    0
 km3L2  km3L   -km3L2  0
-km3   -km3L2   km3    0
 0      0       0      0];

endfunction
```

Listing 5.7: Vektor ekvivalentnog opterećenja od ravnomjerno raspodjeljenog opterećenja na gredi sa otpuštenim momentom kod čvora k, `RavnomjernoOptTiMi_Tk.sci`

```
function rezultat = RavnomjernoOptTiMi_Tk(idx, koordinate,
        elementi, p)
// RavnomjernoOptTiMi_Tk - Vektor
// ekvivalentnog opterecenja na gredi
// sa otpustenim momentom na suprotnom cvoru
// od ravnomjernog opterecenja

x1 = koordinate(elementi(idx, 1), 1);
y1 = koordinate(elementi(idx, 1), 2);
x2 = koordinate(elementi(idx, 2), 1);
y2 = koordinate(elementi(idx, 2), 2);

d = sqrt((x2 - x1)^2 + (y2 - y1)^2);

Mki = -p*d^2/12;

rezultat = [
p*d/2 - 1.5*Mki/d
p*d^2/12 - Mki/2
p*d/2 + 1.5*Mki/d
0];

endfunction
```

Konačno možemo napisati sljedeću skriptu za proračun konstrukcije

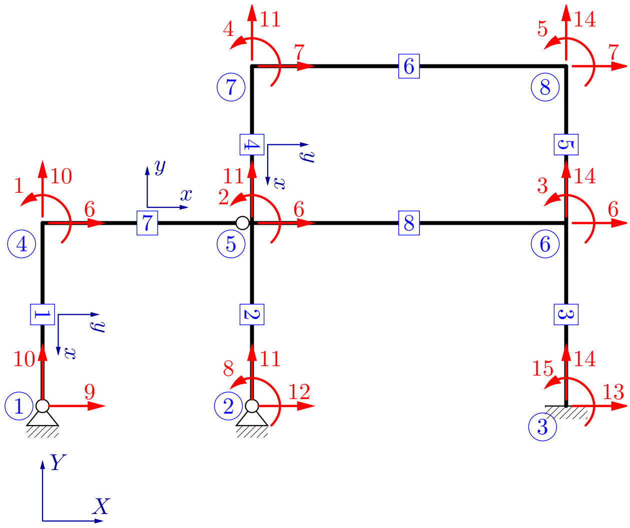
Listing 5.8: Proračun pomjeranja primjenom tehničke metode deformacija, `Zadatak8.sce`

```
// Zadatak 8
// ***************************************************************
clear;
clc;
exec('./Greda2D4x4.sci', -1);
exec('./Greda2DTiMi_Tk.sci', -1);
exec('./RavnomjernoOpt.sci', -1);
exec('./RavnomjernoOptTiMi_Tk.sci', -1);

// ****************** ULAZNI PODACI ************************
// duzina m; sila kN;

//           E       I
presjekG = [3*10^7, 0.3*0.5^3/12];
presjekS = [3*10^7, 0.4^4/12];

// Koordinate cvorova
koordinate = [
0  0
4  0
10 0
0  3.5
4  3.5
10 3.5
4  6.5
10 6.5];

// Definisanje stapova krajnjim cvorovima
elementi =[
1 4
2 5
3 6
5 7
6 8
7 8
4 5
5 6];

SSKEL = [
6 1 0 0
6 2 0 8
6 3 0 0
7 4 6 2
7 5 6 3
0 4 0 5
0 1 0 0
0 2 0 3];

Kels = hypermat([4 4 size(elementi,'r')]);
Fels = hypermat([4 1 size(elementi,'r')]);

Kels(:,:,1)=Greda2DTiMi_Tk(1, koordinate, elementi, presjekS);
Kels(:,:,2) = Greda2D4x4(2, koordinate, elementi, presjekS);
Kels(:,:,3) = Greda2D4x4(3, koordinate, elementi, presjekS);
Kels(:,:,4) = Greda2D4x4(4, koordinate, elementi, presjekS);
Kels(:,:,5) = Greda2D4x4(5, koordinate, elementi, presjekS);
Kels(:,:,6) = Greda2D4x4(6, koordinate, elementi, presjekG);
Kels(:,:,7)=Greda2DTiMi_Tk(7, koordinate, elementi, presjekG);
Kels(:,:,8) = Greda2D4x4(8, koordinate, elementi, presjekG);

Fels(:,:,5) = RavnomjernoOpt(5, koordinate, elementi, -4.0);
Fels(:,:,7)=RavnomjernoOptTiMi_Tk(7,koordinate,elementi,-12);

// ********************* RJESAVANJE ************************

KGlob = zeros(8, 8);
FGlob = zeros(8, 8);

for i=1:size(elementi, 'r')

  Kel = Kels(:,:,i);
  adresa = SSKEL(i,:);

  for j = 1:4
    for k = 1:4
        if (adresa(j) ~= 0) & (adresa(k) ~= 0) then
          KGlob( adresa(j), adresa(k) ) = ..
          KGlob( adresa(j), adresa(k) ) + Kel(j, k);
        end
    end // for k
  end // for j

  //dodavanje vektora opterecenja globalnom vektoru opterecenja
  Fel = Fels(:,:, i);

  for j = 1:4
    if (adresa(j) ~= 0) then
      FGlob( adresa(j) ) = FGlob( adresa(j) ) + Fel(j, 1);
    end
  end // for j

end // for i

// ******** DODAVANJE KONCENTRICNE SILE U CVOROVIMA **********

FGlob( 6 ) = FGlob( 6 ) - 80;

// ************* RJESAVANJE SISTEMA JEDNACINA ****************

UGlob = KGlob \ FGlob;

mprintf("\n================= REZULTATI PRORACUNA");
mprintf("\n=== GLOBALNI VEKTOR POMJERANJA CVOROVA");
mprintf("\n\%15.6f", UGlob(:,1));
```

Listing 5.9: Globalni vektor pomjeranja, `Zadatak2.sce`

```
================= REZULTATI PRORACUNA
=== GLOBALNI VEKTOR POMJERANJA CVOROVA
       0.000438
       0.000489
       0.000914
       0.000247
       0.000066
      -0.005032
      -0.006425
       0.001912
```

Proračun reakcija 5.37



Slika 5.37

Listing 5.10: Zadatak 8A, `Zadatak8A.sce`

```
// Zadatak 8A
// ************************************************************
clear;
clc;
exec('./Greda2D4x4.sci', -1);
exec('./Greda2DTiMi_Tk.sci', -1);
exec('./RavnomjernoOpt.sci', -1);
exec('./RavnomjernoOptTiMi_Tk.sci', -1);

// ****************** ULAZNI PODACI ************************
// duzina m; sila kN;

//            E       I
presjekG = [3*10^7, 0.3*0.5^3/12];
presjekS = [3*10^7, 0.4^4/12];

// Koordinate cvorova
koordinate = [
0  0
4  0
10 0
0  3.5
4  3.5
10 3.5
4  6.5
10 6.5];

// Definisanje stapova krajnjim cvorovima
elementi =[
1 4
2 5
3 6
5 7
6 8
7 8
4 5
5 6];

SSKEL = [
6  1  9  0
6  2 12  8
6  3 13 15
7  4  6  2
7  5  6  3
11 4 14  5
10 1 11  0
11 2 14  3];

Kels = hypermat([4 4 size(elementi,'r')]);
Fels = hypermat([4 1 size(elementi,'r')]);

Kels(:,:,1)=Greda2DTiMi_Tk(1, koordinate, elementi, presjekS);
Kels(:,:,2) = Greda2D4x4(2, koordinate, elementi, presjekS);
Kels(:,:,3) = Greda2D4x4(3, koordinate, elementi, presjekS);
Kels(:,:,4) = Greda2D4x4(4, koordinate, elementi, presjekS);
Kels(:,:,5) = Greda2D4x4(5, koordinate, elementi, presjekS);
Kels(:,:,6) = Greda2D4x4(6, koordinate, elementi, presjekG);
Kels(:,:,7)=Greda2DTiMi_Tk(7, koordinate, elementi, presjekG);
Kels(:,:,8) = Greda2D4x4(8, koordinate, elementi, presjekG);

Fels(:,:,5) = RavnomjernoOpt(5, koordinate, elementi, -4.0);
Fels(:,:,7)=RavnomjernoOptTiMi_Tk(7,koordinate,elementi,-12);

// ********************* RJESAVANJE ************************
KGlob = zeros(15, 15);
FGlob = zeros(15, 15);

for i=1:size(elementi, 'r')
  Kel = Kels(:,:,i);
  adresa = SSKEL(i,:);

  for j = 1:4
    for k = 1:4
        if (adresa(j) ~= 0) & (adresa(k) ~= 0) then
          KGlob( adresa(j), adresa(k) ) = ..
          KGlob( adresa(j), adresa(k) ) + Kel(j, k);
        end
    end // for k
  end // for j

//dodavanje vektora opterecenja globalnom vektoru opterecenja
  Fel = Fels(:,:, i);

  for j = 1:4
        if (adresa(j) ~= 0) then
          FGlob( adresa(j) ) = FGlob( adresa(j) ) + Fel(j, 1);
        end
  end // for j
end // for i

// ******** DODAVANJE KONCENTRICNE SILE U CVOROVIMA **********

FGlob( 6 ) = FGlob( 6 ) - 80;

// ************* RJESAVANJE SISTEMA JEDNACINA ****************
// Nema pomjeranja oslonaca pa se za rjesavanje
// sistema jednacina uzima "gornji lijevi" blok matrice
// krutosti i odgovarajuci vektor sila

K11 = KGlob(1:8,1:8);
F1 = FGlob(1:8,1);

// RJESENJE SISTEMA JEDNACINA
U1 = K11 \ F1;

UGlob = zeros(15, 15);
UGlob(1:8, 1) = U1(1:8, 1);

mprintf("\n================= REZULTATI PRORACUNA");
mprintf("\n=== GLOBALNI VEKTOR POMJERANJA CVOROVA");
mprintf("\n\%15.6f", UGlob(1:15,1));

// ************* PRORACUN REAKCIJA ***************************

Reakc = KGlob * UGlob - FGlob;

mprintf("\n============= GLOBALNI VEKTOR REAKCIJA");
mprintf("\n%11.2f", Reakc(1:15,1));

// **************** POST PROCESSING *************************
// *** PRORACUN POMJERANJA I PRESJECNIH SILA PO ELEMENTIMA ***
for i = 1:size(elementi,'r')
  mprintf('\n=========================== STAP %d', i);

  adresa = SSKEL(i,:);

  // uzimanje pomjeranja stapa iz globalnog vektora pomjeranja
  for j = 1:4
      if adresa(j) ~= 0 then
          u( j ) = UGlob( adresa(j) );
      else
          u( j ) = 0;
      end;
```

```
    end // for j

    // printanje vektora pomjeranja po stapovima
    mprintf("\nVektor pomjeranja");
    mprintf("\n%15.6f", u(1:4,1));

    // U opstem slucaju vektor pomjeranja se mora transformisati
    // iz globalnog u lokalni koordinatni sistem, ali ovdje
    // lokalni sistem nije zarotiran
         // (jer smo tako formirali SSKEL polje)
         // pa je u.l = T' * U.g

    // matrica krutosti i vektor vanjskog opterecenja
    Ke = Kels(:,:,i);
    Fe = Fels(:,:,i);

    // proracun vektora sila na stapu
    f = Ke * u - Fe;

    mprintf("\nVektor Sila");
    mprintf("\n\%11.2f", f(1:4,1));
  end // for i
```

Listing 5.11: Rezultat proračuna, `Zadatak8A.sce`

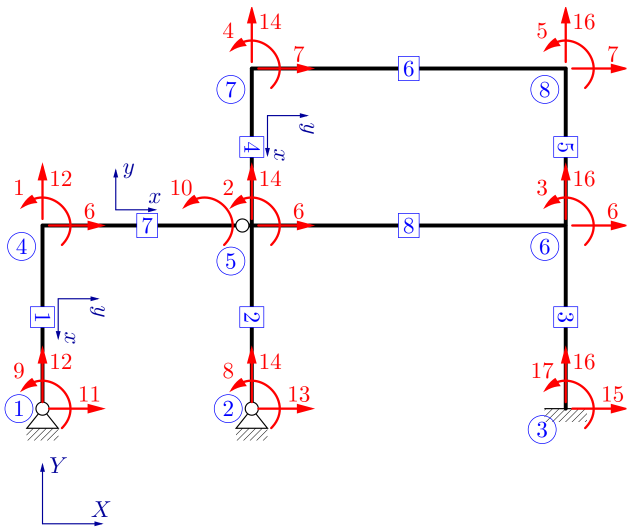
```
630 ================= REZULTATI PRORACUNA
631 === GLOBALNI VEKTOR POMJERANJA CVOROVA
632        0.000438
633        0.000489
634        0.000914
635        0.000247
636        0.000066
637       -0.005032
638       -0.006425
639        0.001912
640        0.000000
641        0.000000
642        0.000000
643        0.000000
644        0.000000
645        0.000000
646        0.000000
647 ============ GLOBALNI VEKTOR REAKCIJA
648        0.00
649       -0.00
650        0.00
651        0.00
652        0.00
653        0.00
654       -0.00
655        0.00
656       15.66
657       37.71
658       37.11
659       14.87
660       61.47
661      -26.81
662     -124.30
663 =========================== STAP 1
664 Vektor pomjeranja
665       -0.005032
666        0.000438
667        0.000000
668        0.000000
669 Vektor Sila
670      -15.66
671      -54.82
672       15.66
673        0.00
674 =========================== STAP 2
675 Vektor pomjeranja
676       -0.005032
677        0.000489
678        0.000000
679        0.001912
680 Vektor Sila
681      -14.87
682      -52.03
683       14.87
684        0.00
685 =========================== STAP 3
686 Vektor pomjeranja
687       -0.005032
688        0.000914
689        0.000000
690        0.000000
691 Vektor Sila
692      -61.47
693      -90.85
694       61.47
695     -124.30
696 =========================== STAP 4
697 Vektor pomjeranja
698       -0.006425
699        0.000247
700       -0.005032
701        0.000489
702 Vektor Sila
703       -8.20
704      -17.47
705        8.20
706       -7.13
707 =========================== STAP 5
708 Vektor pomjeranja
709       -0.006425
710        0.000066
711       -0.005032
712        0.000914
713 Vektor Sila
714        8.20
715      -11.81
716        3.80
717       18.41
718 =========================== STAP 6
719 Vektor pomjeranja
720        0.000000
721        0.000247
722        0.000000
723        0.000066
724 Vektor Sila
725        4.88
726       17.47
727       -4.88
728       11.81
729 =========================== STAP 7
730 Vektor pomjeranja
731        0.000000
732        0.000438
733        0.000000
734        0.000000
735 Vektor Sila
736       37.71
737       54.82
738       10.29
739        0.00
740 =========================== STAP 8
741 Vektor pomjeranja
742        0.000000
743        0.000489
744        0.000000
745        0.000914
746 Vektor Sila
747       21.93
748       59.16
749      -21.93
750       72.44
```

Primijetimo da se i za elemente kojima je otpušten momenat na kraju printa vektor 4×1 iako takav štap ima 3 stepena slobode kretanja. Potrebno je znati dakle da vrijednost na liniji 668 ne predstavlja rotaciju kraja štapa, vec ćemo u ovakvom slučaju morati proračunati rotaciju kraja štapa kako slijedi.

$$
\begin{aligned}
\varphi_k &= -0.5\varphi_i + 1.5\frac{\Delta_{ik}}{l_{ik}} + 0.25\frac{M^{\bullet}_{ki}}{k_{ik}}, \quad (\Delta_{ik} = v_k - v_i) \\
&= -0.5 \cdot 0.000438 + 1.5 \cdot \frac{0 - (-0.005032)}{3.5} \\
&= 0.001938
\end{aligned}
\tag{5.10}
$$

Ovo bismo morali uraditi na svakom mjestu gdje smo statičkom kondenzacijom izbacili stepen slobode kretanja. U sljedećoj vježbi ćemo formirati sistem jednačina tako što ćemo uvesti stepen slobode kretanja na svako mjesto gdje želimo proračunati pomjeranje na konstrukciji. Stepeni slobode kretanja su dati na slici 5.38. Možemo vidjeti da smo definisali SSK 10 da bismo osigurali različito pomjeranje kraja štapa 7 od ostalih štapova vezanih u čvoru 5 čime modeliramo zglobnu vezu štapa 7. U ovom slučaju za štap 7 koristimo matricu kruto vezanog štapa na oba kraja.

Proračun svih pomjeranja 5.38



Slika 5.38

Listing 5.12: Zadatak 8B, `Zadatak8B.sce`

```
// Zadatak 8B
// ************************************************************
clear;
clc;
exec('./Greda2D4x4.sci', -1);
exec('./RavnomjernoOpt.sci', -1);

// ****************** ULAZNI PODACI ************************
// duzina m; sila kN;

//            E       I
presjekG = [3*10^7, 0.3*0.5^3/12];
presjekS = [3*10^7, 0.4^4/12];

// Koordinate cvorova
koordinate = [
0  0
4  0
10 0
0  3.5
4  3.5
10 3.5
4  6.5
10 6.5];

// Definisanje stapova krajnjim cvorovima
elementi =[
1 4
2 5
3 6
5 7
6 8
7 8
4 5
5 6];

SSKEL = [
6  1 11  9
6  2 13  8
6  3 15 17
7  4  6  2
7  5  6  3
14  4 16  5
12  1 14 10
14  2 16  3];

Kels = hypermat([4 4 size(elementi,'r')]);
Fels = hypermat([4 1 size(elementi,'r')]);

Kels(:,:,1) = Greda2D4x4(1, koordinate, elementi, presjekS);
Kels(:,:,2) = Greda2D4x4(2, koordinate, elementi, presjekS);
Kels(:,:,3) = Greda2D4x4(3, koordinate, elementi, presjekS);
Kels(:,:,4) = Greda2D4x4(4, koordinate, elementi, presjekS);
Kels(:,:,5) = Greda2D4x4(5, koordinate, elementi, presjekS);
Kels(:,:,6) = Greda2D4x4(6, koordinate, elementi, presjekG);
Kels(:,:,7) = Greda2D4x4(7, koordinate, elementi, presjekG);
Kels(:,:,8) = Greda2D4x4(8, koordinate, elementi, presjekG);

Fels(:,:,5) = RavnomjernoOpt(5, koordinate, elementi, -4.0);
Fels(:,:,7) = RavnomjernoOpt(7, koordinate, elementi, -12.0);

// ********************** RJESAVANJE ************************
KGlob = zeros(17, 17);
FGlob = zeros(17, 17);

for i=1:size(elementi, 'r')

  Kel = Kels(:,:,i);
  adresa = SSKEL(i,:);

  for j = 1:4
    for k = 1:4
        if (adresa(j) ~= 0) & (adresa(k) ~= 0) then
          KGlob( adresa(j), adresa(k) ) = ..
          KGlob( adresa(j), adresa(k) ) + Kel(j, k);
        end
    end // for k
  end // for j

//dodavanje vektora opterecenja globalnom vektoru opterecenja
  Fel = Fels(:,:, i);
  for j = 1:4
        if (adresa(j) ~= 0) then
          FGlob( adresa(j) ) = FGlob( adresa(j) ) + Fel(j, 1);
        end
  end // for j
end // for i

// ******** DODAVANJE KONCENTRICNE SILE U CVOROVIMA **********

FGlob( 6 ) = FGlob( 6 ) - 80;

// ************* RJESAVANJE SISTEMA JEDNACINA ****************
// Nema pomjeranja oslonaca pa se za rjesavanje
// sistema jednacina uzima "gornji lijevi" blok matrice
// krutosti i odgovarajuci vektor sila

K11 = KGlob(1:10,1:10);
F1 = FGlob(1:10,1);

// rjesenje
U1 = K11 \ F1;

UGlob = zeros(17, 17);
UGlob(1:10, 1) = U1(1:10, 1);

mprintf("\n================== REZULTATI PRORACUNA");
mprintf("\n=== GLOBALNI VEKTOR POMJERANJA CVOROVA");
mprintf("\n\%15.6f", UGlob(1:17,1));

// ************* PRORACUN REAKCIJA ***************************

Reakc = KGlob * UGlob - FGlob;

mprintf("\n============= GLOBALNI VEKTOR REAKCIJA");
mprintf("\n%11.2f", Reakc(1:17,1));

// **************** POST PROCESSING *************************
// *** PRORACUN POMJERANJA I PRESJECNIH SILA PO ELEMENTIMA ***
```

```

for i = 1:size(elementi,'r')
  mprintf('\n=========================== STAP %d', i);

  adresa = SSKEL(i,:);

  // uzimanje pomjeranja stapa iz globalnog vektora pomjeranja
  for j = 1:4
      if adresa(j) ~= 0 then
          u( j ) = UGlob( adresa(j) );
      else
          u( j ) = 0;
      end;
  end // for j

  // printanje vektora pomjeranja po stapovima
  mprintf("\nVektor pomjeranja");
  mprintf("\n%15.6f", u(1:4,1));

  // U opstem slucaju vektor pomjeranja se mora transformisati
  // iz globalnog u lokalni koordinatni sistem, ali ovdje
  // lokalni sistem nije zarotiran
        // (jer smo tako formirali SSKEL polje)
        // pa je u.l = T' * U.g

  // matrica krutosti i vektor vanjskog opterecenja
  Ke = Kels(:,:,i);
  Fe = Fels(:,:,i);

  // proracun vektora sila na stapu
  f = Ke * u - Fe;

  mprintf("\nVektor Sila");
  mprintf("\n\%11.2f", f(1:4,1));
end // for i
```

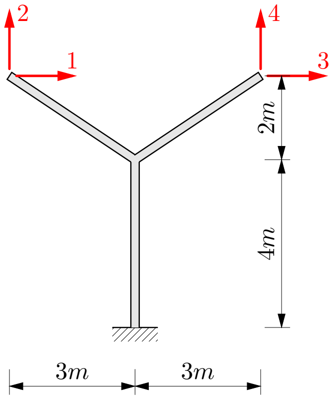
Listing 5.13: Rezultat proračuna - Zadatak 8B, `Zadatak8B.sce`

```
================= REZULTATI PRORACUNA
=== GLOBALNI VEKTOR POMJERANJA CVOROVA
       0.000438
       0.000489
       0.000914
       0.000247
       0.000066
      -0.005032
      -0.006425
       0.001912
       0.001937
      -0.000049
       0.000000
       0.000000
       0.000000
       0.000000
       0.000000
       0.000000
       0.000000
============ GLOBALNI VEKTOR REAKCIJA
      -0.00
       0.00
       0.00
       0.00
      -0.00
       0.00
      -0.00
       0.00
       0.00
      -0.00
      15.66
      37.71
      14.87
      37.11
      61.47
     -26.81
    -124.30
=========================== STAP 1
Vektor pomjeranja
      -0.005032
       0.000438
       0.000000
       0.001937
Vektor Sila
      -15.66
      -54.82
       15.66
        0.00
=========================== STAP 2
Vektor pomjeranja
      -0.005032
       0.000489
       0.000000
       0.001912
Vektor Sila
      -14.87
      -52.03
       14.87
        0.00
=========================== STAP 3
Vektor pomjeranja
      -0.005032
       0.000914
       0.000000
       0.000000
Vektor Sila
      -61.47
      -90.85
       61.47
     -124.30
=========================== STAP 4
Vektor pomjeranja
      -0.006425
       0.000247
      -0.005032
       0.000489
Vektor Sila
       -8.20
      -17.47
        8.20
       -7.13
=========================== STAP 5
Vektor pomjeranja
      -0.006425
       0.000066
      -0.005032
       0.000914
Vektor Sila
        8.20
      -11.81
        3.80
       18.41
=========================== STAP 6
Vektor pomjeranja
       0.000000
       0.000247
       0.000000
       0.000066
Vektor Sila
        4.88
       17.47
       -4.88
       11.81
=========================== STAP 7
Vektor pomjeranja
       0.000000
       0.000438
       0.000000
      -0.000049
Vektor Sila
       37.71
       54.82
       10.29
       -0.00
=========================== STAP 8
Vektor pomjeranja
       0.000000
       0.000489
       0.000000
       0.000914
Vektor Sila
       21.93
       59.16
      -21.93
       72.44
```

Jednačinom 5.10 smo u prethodnom postupku proračunali rotaciju kraja štapa 1 i dobili $\varphi_k = 0.001938$, dok smo sad uveli SSK 9 pa je pomjeranje $\boldsymbol{u}_9 = 0.001937$ dato na liniji 915

Zadatak 9 Formirati matricu krutosti makroelementa prikazanog na slici 5.39. Makroelement treba da ima stepene slobode kretanja naznačene na slici 5.39. Takođe, uzeti u obzir i poznata pomjeranja čvorova u osloncima.

$$E = 3.0 \cdot 10^7 kN/m^2$$
$$A = 2.0 m^2$$
$$I = 1.667 \cdot 10^{-1} m^4$$



Slika 5.39

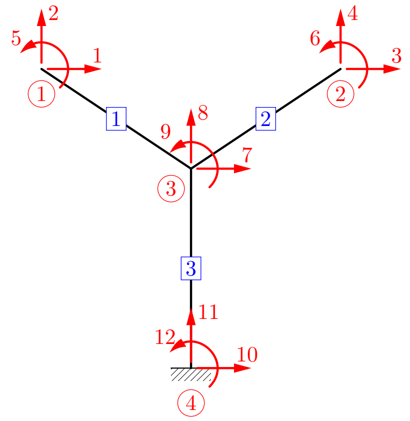
Rješenje Formiraćemo matricu krutosti konstrukcije a onda statičkom kondenzacijom dobiti matricu krutosti izraženu preko traženih stepeni slobode kretanja. Na slici 5.40 data je numeracija čvorova, štapova i SSK.

Listing 5.14: Formiranje matrice krutosti u lokalnom koordinatnom sistemu, `Greda2D.sce`

```
function rezultat = Greda2D(x1,y1,x2,y2,Em,A,I)
  // Greda2D - Matrica krutosti ravnog grednog stapa

  duzina = sqrt((x2 - x1)^2 + (y2 - y1)^2);

  ka = Em * A / duzina;
  km12 = 12 * Em * I / duzina^3;
  km6= km12 * duzina / 2;
  km4= km6 * duzina / 1.5;
  km2= km4 / 2;

  rezultat = [
   ka      0      0     -ka     0        0
   0      km12   km6     0     -km12    km6
   0      km6    km4     0     -km6     km2
  -ka      0      0      ka     0        0
   0     -km12  -km6     0      km12   -km6
   0      km6    km2     0     -km6     km4];
endfunction
```

Listing 5.15: Formiranje matrice transformacije, `Transformacija.sce`

```
function rezultat = Transformacija(x1,y1,x2,y2)
  // Transformacija - Matrica transformacije ravnog grednog stapa

  duzina = sqrt((x2 - x1)^2 + (y2 - y1)^2);

  c = (x2 - x1)/duzina;
  s = (y2 - y1)/duzina;

  rezultat = [
    c    s    0    0    0     0
   -s    c    0    0    0     0
    0    0    1    0    0     0
    0    0    0    c    s     0
    0    0    0   -s    c     0
    0    0    0    0    0     1];
endfunction
```



Slika 5.40

Listing 5.16: Proračun matrice krutosti makroelementa, `StatKond.sce`

```
// Zadatak 9
// ***********************************************************

exec('./Greda2D.sce', -1);
exec('./Transformacija.sce', -1);

// ****************** ULAZNI PODACI ************************
// duzina m; sila kN

E = 3*10^7;
A = 2;
I = 0.1667;

// Koordinate cvorova
cvor = [
-3 2
3 2
0 0
0 -4];

// Definisanje stapova krajnjim cvorovima
stap =[
1 3
3 2
3 4];

// Stepeni slobode kretanja po cvorovima
SSK = [
1 2 5
3 4 6
7 8 9
10 11 12];

// ******************** RJESENJE ***************************

KGlob = zeros(12, 12);

for i = 1:size(stap,'r') // size(stap,'r') = BrojStapova

  ni = stap(i, 1);
  nj = stap(i, 2);

  x1 = cvor(ni, 1);
  y1 = cvor(ni, 2);

  x2 = cvor(nj, 1);
  y2 = cvor(nj, 2);

  Ke = Greda2D(x1, y1, x2, y2, E, A, I);
  T = Transformacija(x1, y1, x2, y2);
  Kg = T` * Ke * T;

  adresa(1) = SSK(ni, 1);
  adresa(2) = SSK(ni, 2);
  adresa(3) = SSK(ni, 3);
  adresa(4) = SSK(nj, 1);
  adresa(5) = SSK(nj, 2);
  adresa(6) = SSK(nj, 3);
```

```

  for j = 1:6
    for k = 1:6
      KGlob( adresa(j), adresa(k) ) = ..
      KGlob( adresa(j), adresa(k) ) + Kg(j, k);
    end // for k
  end // for j

end //for i za sve stapove

//Staticka kondenzacija za izrazavanje matrice krutosti
//preko prva cetiri stepena slobode kretanja
KStub = KGlob(1:4,1:4) - KGlob(1:4,5:9) * inv(KGlob(5:9,5:9))
      * KGlob(5:9,1:4)
```

Rezultat proračuna je matrica krutosti makroelementa:

Listing 5.17: Rezultat - matrica krutosti makroelementa.

```
  3761215.817 -2399657.769 -3439458.914 -1999650.445
 -2399657.769  1838708.033  1999650.445  1281100.938
 -3439458.914  1999650.445  3761215.817  2399657.769
 -1999650.445  1281100.938  2399657.769  1838708.033
```

Zadatak 10 Odrediti reakcije, pomjeranja čvorova i presječne sile za konstrukciju prikazanu na slici 5.41. Kako su karakteristike materijala i poprečnog presjeka štapova stuba iste kao u prethodnom zadatku, pri formiranju matrice krutosti sistema ne proračunavati matrice krutosti štapova stuba već iskoristiti matricu krutosti makroelementa iz prethodnog zadatka.

Rješenje Numeracija čvorova, štapova i SSK je data na slici 5.42.

Na liniji 1239 unijeli smo matricu krutosti makro elementa proračunatu u prethodnom zadatku. Na liniji 1245 formiramo polja stepeni slobode kretanja stubova, a zatim na liniji 1249 formiramo petlju kojom dodajemo matrice krutosti makroelemenata (stubova) globalnoj matrici krutosti.

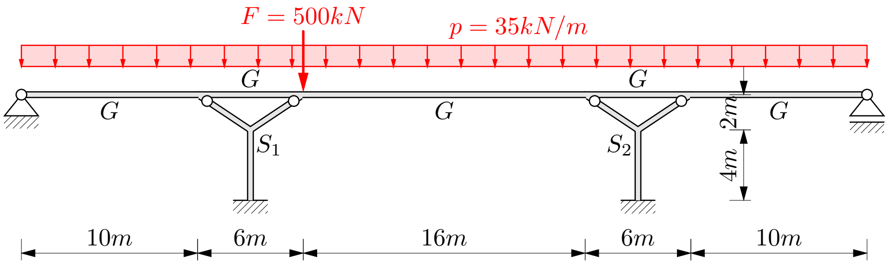
Listing 5.18: Proračun vektora ekvivalentnog opterećenja na grednom elementu od ravnomjerno raspodjeljenog opterećenja po cijelom štapu, `JednolikoGreda2D.sce`

```
function rezultat = JednolikoGreda2D(x1,y1,x2,y2,p)
  // JednolikoGreda2D - Vektor ekvivalentnog opterecenja na
       gredi
  // od ravnomjernog opterecenja

  d = sqrt((x2 - x1)^2 + (y2 - y1)^2);

  rezultat = [0 p*d/2 p*d^2/12 0  p*d/2 -p*d^2/12];
endfunction
```

Sa ovim možemo napisati proceduru koja formira globalnu matricu krutosti i vektor opterećenja konstrukcije prikazane na slici 5.41

Listing 5.19: Formiranje globalne matrice krutosti i vektora opterećenja za konstrukciju na slici 5.41, `Most.sce`

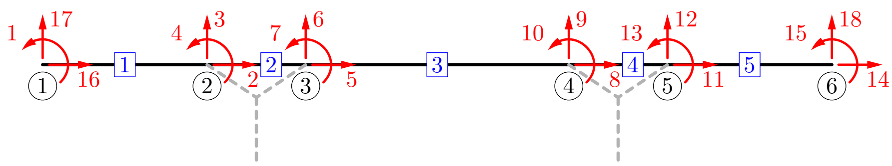
```
1148 // Zadatak 10 **************************************************
1149 clear;
1150 clc;
1151
1152 exec('./Greda2D.sce', -1);
1153 exec('./JednolikoGreda2D.sce', -1);
1154 exec('./Transformacija.sce', -1);
1155
1156 // ********************* ULAZNI PODACI **********************
1157 // duzina m; sila kN;
1158
1159 E = 3*10^7;
1160 A = 1;
1161 I = 0.02083;
1162
1163 // Koordinate cvorova
1164 cvor = [
1165 0 0
1166 10 0
1167 16 0
1168 32 0
1169 38 0
1170 48 0];
1171
1172 // Definisanje stapova krajnjim cvorovima
1173 stap =[
1174 1 2
1175 2 3
1176 3 4
1177 4 5
1178 5 6];
1179
1180 // Opterecenje po stapovima
1181 p = [
1182 -35
1183 -35
1184 -35
1185 -35
1186 -35];
1187
1188 // Stepeni slobode kretanja po cvorovima
1189 SSK = [
1190 16 17 1
1191 2 3 4
1192 5 6 7
1193 8 9 10
1194 11 12 13
1195 14 18 15];
1196
1197 // ******************** RJESAVANJE *************************
1198
1199 KGlob = zeros(18, 18);
1200 FGlob = zeros(18, 18);
1201
1202 for i = 1:size(stap,'r') // size(stap,'r') = BrojStapova
1203
1204   ni = stap(i, 1);
1205   nj = stap(i, 2);
1206
1207   x1 = cvor(ni, 1);
1208   y1 = cvor(ni, 2);
1209
1210   x2 = cvor(nj, 1);
1211   y2 = cvor(nj, 2);
1212
1213   Ke = Greda2D(x1, y1, x2, y2, E, A, I);
1214   T = Transformacija(x1, y1, x2, y2);
1215   Kg = T` * Ke * T;
1216
1217   F = JednolikoGreda2D(x1, y1, x2, y2, p(i));
1218
1219   adresa(1) = SSK(ni, 1);
1220   adresa(2) = SSK(ni, 2);
1221   adresa(3) = SSK(ni, 3);
1222   adresa(4) = SSK(nj, 1);
1223   adresa(5) = SSK(nj, 2);
1224   adresa(6) = SSK(nj, 3);
1225
1226   for j = 1:6
1227     for k = 1:6
1228       KGlob( adresa(j), adresa(k) ) = ..
1229       KGlob( adresa(j), adresa(k) ) + Kg(j, k);
1230     end // for k
1231
1232     FGlob( adresa(j) ) = FGlob( adresa(j) ) + F(j);
1233   end // for j
1234
1235 end //for i za sve stapove
1236
1237 // Matrica krutosti stubova je vec proracunata
1238 // u prethodnom zadatku
1239 KStub = [
1240    3761215.8 -2399657.8 -3439458.9 -1999650.4
1241   -2399657.8  1838708.0  1999650.4  1281100.9
1242   -3439458.9  1999650.4  3761215.8  2399657.8
1243   -1999650.4  1281100.9  2399657.8  1838708.0];
1244 // Dodavanje matrice krutosti stubova
1245 SSK_STUB = [
1246 2 3 5 6
1247 8 9 11 12];
1248
1249 for i = 1:2 //stuba
1250   for j = 1:4
1251     for k = 1:4
1252       KGlob( SSK_STUB(i,j), SSK_STUB(i,k) ) = ..
1253       KGlob( SSK_STUB(i,j), SSK_STUB(i,k) ) + KStub(j, k);
1254      end // for k
1255   end // for j
1256 end
1257
1258 // ******* DODAVANJE KONCENTRICNE SILE U CVOROVIMA ***********
1259 FGlob(6,1) = FGlob(6,1) -500;
1260
1261 // ************ RJESAVANJE SISTEMA JEDNACINA *****************
1262
1263 // Nema pomjeranja fiksnih oslonaca pa se za rjesavanje
1264 // sistema jednacina uzima "gornji lijevi" blok matrice
1265 // krutosti i odgovarajuci vektor sila
1266
1267 K11 = KGlob(1:15,1:15);
1268 F1 = FGlob(1:15,1);
1269
1270 // rjesenje
1271 U1 = K11 \ F1;
1272
1273 UGlob = zeros(18, 18);
1274 UGlob(1:15, 1) = U1(1:15, 1);
```



G :
$E = 3 \cdot 10^7 kN/m^2$
$A = 1.0m^2$
$I = 0.02083m^4$

$S_{1,2}$:
$E = 3 \cdot 10^7 kN/m^2$
$A = 2.0m^2$
$I = 0.1667m^4$

Slika 5.41



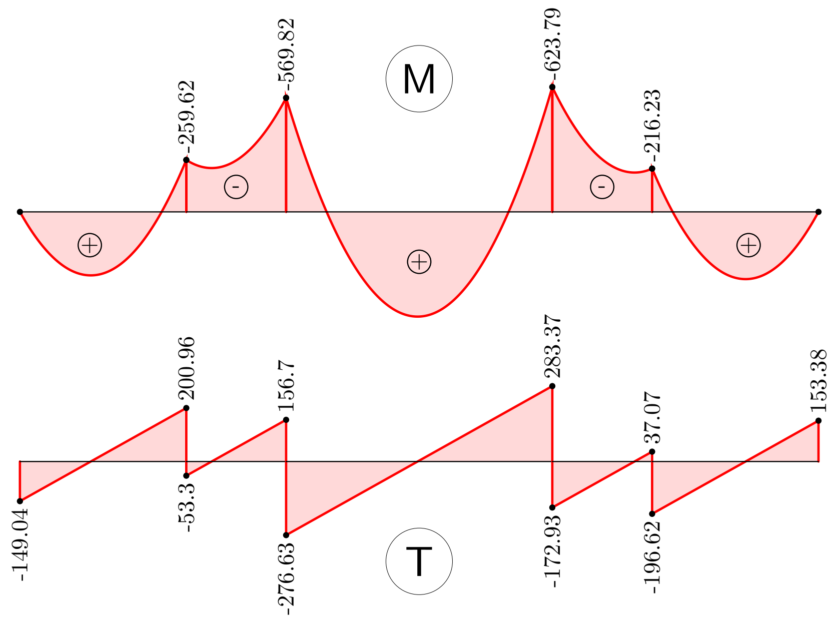
Slika 5.42

```
mprintf("\n================== REZULTATI PRORACUNA");
mprintf("\n=== GLOBALNI VEKTOR POMJERANJA CVOROVA");
mprintf("\n%15.6f", UGlob(1:18,1));

// ************ PRORACUN REAKCIJA ****************************

Reakc = KGlob * UGlob - FGlob;

mprintf("\n============= GLOBALNI VEKTOR REAKCIJA");
mprintf("\n%11.2f", Reakc(16:18,1));


// *************** POST PROCESSING ***************************

for i = 1:size(stap,'r') // size(stap,'r') = BrojStapova
  mprintf('\n=========================== STAP %d', i);
  ni = stap(i, 1);
  nj = stap(i, 2);

  x1 = cvor(ni, 1);
  y1 = cvor(ni, 2);

  x2 = cvor(nj, 1);
  y2 = cvor(nj, 2);

  adresa(1) = SSK(ni, 1);
  adresa(2) = SSK(ni, 2);
  adresa(3) = SSK(ni, 3);
  adresa(4) = SSK(nj, 1);
  adresa(5) = SSK(nj, 2);
  adresa(6) = SSK(nj, 3);

  // uzimanje pomjeranja stapa iz globalnog vektora pomjeranja
  for j = 1:6
    u( j ) = UGlob( adresa(j) );
  end // for j

  // printanje vektora pomjeranja po stapovima
  mprintf("\nVektor pomjeranja");
  mprintf("\n%15.6f", u(1:6,1));

  // U opstem slucaju vektor pomjeranja se mora
        // transformisati iz globalnog u lokalni
  // koordinatni sistem, ali ovdje lokalni
  // sistem nije zarotiran pa je u = T' * Ug

  // matrica krutosti i vektor vanjskog opterecenja
  Ke = Greda2D(x1, y1, x2, y2, E, A, I);
  F = JednolikoGreda2D(x1, y1, x2, y2, p(i));

  // proracun vektora sila na stapu
  f = Ke * u - F;

  mprintf("\nVektor Sila");
  mprintf("\n%11.2f", f(1:6,1));


end // for i
```

Dobija se rješenje za globalni vektor pomjeranja i reakcija

$$\boldsymbol{u}_{glob} = \begin{bmatrix} -0.001598 \\ 0.000100 \\ 0.000434 \\ 0.000992 \\ 0.000264 \\ -0.001045 \\ -0.001982 \\ 0.000171 \\ -0.000213 \\ 0.001855 \\ 0.000244 \\ -0.000111 \\ -0.001169 \\ 0.000244 \\ 0.001768 \\ 0.000000 \\ 0.000000 \\ 0.000000 \end{bmatrix} \qquad \boldsymbol{r}_{glob} = \begin{bmatrix} 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ 0.00 \\ -300.35 \\ 149.04 \\ 153.38 \end{bmatrix} \tag{5.11}$$

Kao i vektori pomjeranja i sila po štapovima



Slika 5.43

Vektor	Štap 1	Štap 2	Štap 3
u_i	0.000000	0.000100	0.000264
v_i	0.000000	0.000434	-0.001045
φ_i	-0.001598	0.000992	-0.001982
u_j	0.000100	0.000264	0.000171
v_j	0.000434	-0.001045	-0.000213
φ_j	0.000992	-0.001982	0.001855
N_i	-300.35	-818.79	174.12
T_i	149.04	53.30	276.63
M_i	0.00	259.62	569.82
N_j	300.35	818.79	-174.12
T_j	200.96	156.70	283.37
M_j	-259.62	-569.82	-623.79

Tablica 5.1: Rezultat proračuna po štapovima

Vektor	Štap 4	Štap 5
u_i	0.000171	0.000244
v_i	-0.000213	-0.000111
φ_i	0.001855	-0.001169
u_j	0.000244	0.000244
v_j	-0.000111	0.000000
φ_j	-0.001169	0.001768
N_i	-363.21	-0.00
T_i	172.93	196.62
M_i	623.79	216.23
N_j	363.21	0.00
T_j	37.07	153.38
M_j	-216.23	0.00

Tablica 5.2: Rezultat proračuna po štapovima