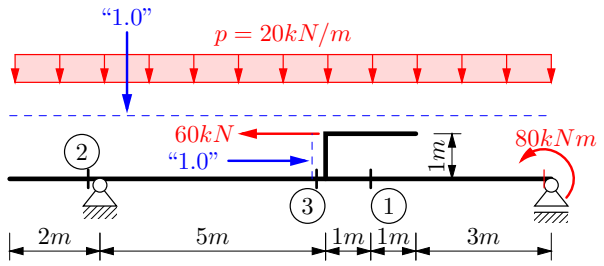
Zadatak 4 Za nosač i opterećenje na slici 2.6:

a) nacrtati dijagrame momenata, transverzalnih i normalnih sila,

b) nacrtati uticajne linije za presječne sile u naznačenim presjecima,

c) rezultate dobijene pod a) kontrolisati preko uticajnih linija dobijenih pod b)

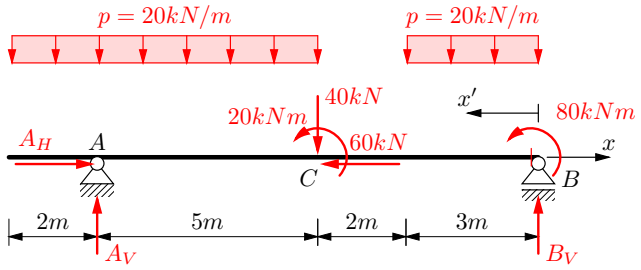


Slika 2.6

Rješenje

a) Dijagrami presječnih sila

Određivanje reakcija



Slika 2.7

$\rightarrow$: $A_H = 60kN$

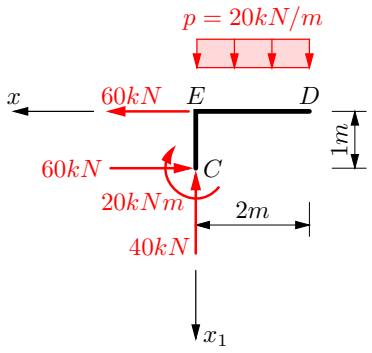
$\curvearrowleft A$: $7 \cdot 20 \cdot 1.5 + 40 \cdot 5 - 20 + 60 \cdot 8.5 - 80 - B_V \cdot 10 = 0$

$\Rightarrow B_V = 82kN$

$\curvearrowleft B$: $A_V \cdot 10 - 140 \cdot 8.5 - 40 \cdot 5 - 20 - 60 \cdot 1.5 - 80 = 0$

$\Rightarrow A_V = 158kN$

Provjera $\uparrow$: $158 + 82 - 40 - 10 \cdot 20 = 0$



Slika 2.8

Presječne sile

Dio D-E

$0 \leq x < 2$

$M(x) = -20 \cdot x \cdot \frac{x}{2} = -10x^2$

$T(x) = -20x$

$N(x) = 0$

Dio E-C

$0 \leq x_1 \leq 1$

$M(x_1) = -40 + 60 \cdot x_1$

$T(x_1) = 60$

$N(x_1) = -40$

Dio A-B

$0 \leq x \leq 2$

$M(x) = -20 \cdot x \cdot \frac{x}{2} = -10x^2$

$T(x) = 20x$

$N(x) = 0$

x	0	2
M	0	-40
T	0	40
N	0	0

$2 \leq x \leq 7$

$M(x) = -20 \cdot x \cdot \frac{x}{2} + 158 \cdot (x-2) = -10x^2 + 158x - 316$

$T(x) = 20x - 158$

$N(x) = -60$

x	2	7
M	-40	300
T	-118	-18
N	-60	-60

$0 \leq x' \leq 3$

$M(x') = 80 + 82 \cdot x' - 20 \cdot x' \cdot \frac{x'}{2} = -10x'^2 + 82x' + 80$

$T(x') = 82 - 20x'$

$N(x') = 0$

x'	0	3
M	80	236
T	82	22
N	0	0

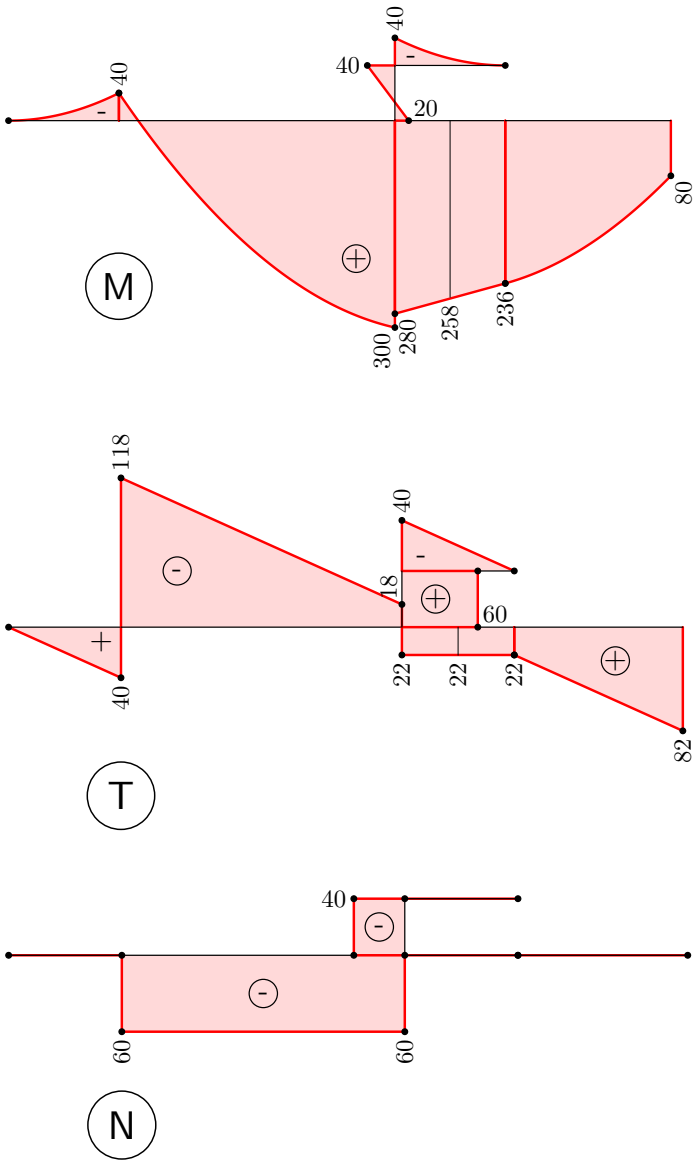
$3 \leq x' \leq 5$

$M(x') = 80 + 82 \cdot x' - 60(x' - 1.5) = 22x' + 170$

$T(x') = 82 - 3 \cdot 20 = 22$

$N(x') = 0$

x'	3	4	5
M	236	258	280
T	22	22	22
N	0	0	0



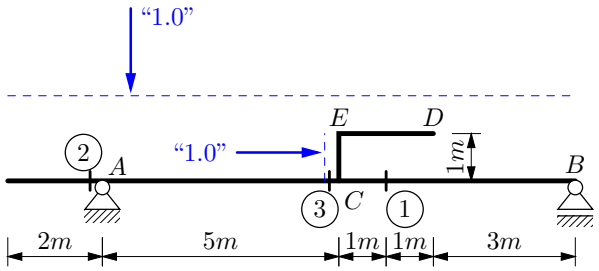
b) Uticajne linije
Vertikalna pokretna sila "1"

$\overset{\curvearrowleft}{A}: \quad B_V \cdot 10 - 1.0 \cdot (x-2) = 0$

$$\Rightarrow B_V = \frac{x-2}{10}$$

$\overset{\curvearrowright}{B}: \quad A_V \cdot 10 - 1.0 \cdot (12-x) = 0$

$$\Rightarrow A_V = \frac{12-x}{10}$$



Slika 2.9

Presjek 1

0 ↓ *D*

$$M_1 = 4 \cdot B_V = 4 \cdot \frac{x-2}{10}$$

$$T_1 = B_V = \frac{x-2}{10}$$

$$N_1 = 0$$

D* ↓ *B

$$M_1 = 6 \cdot A_V = 6 \cdot \frac{12-x}{10}$$

$$T_1 = -A_V = \frac{x-12}{10}$$

$$N_1 = 0$$

Presjek 2

0 ↓ 2

$$M_2 = -1.0 \cdot (2-x) = x-2$$

$$T_2 = 1.0$$

$$N_2 = 0$$

2 ↓ *B*

$$M_2 = 0$$

$$T_2 = 0$$

$$N_2 = 0$$

Presjek 3

0 ↓ 3

$$M_3 = 5 \cdot B_V = \frac{x-2}{2}$$

$$T_3 = B_V = \frac{x-2}{10}$$

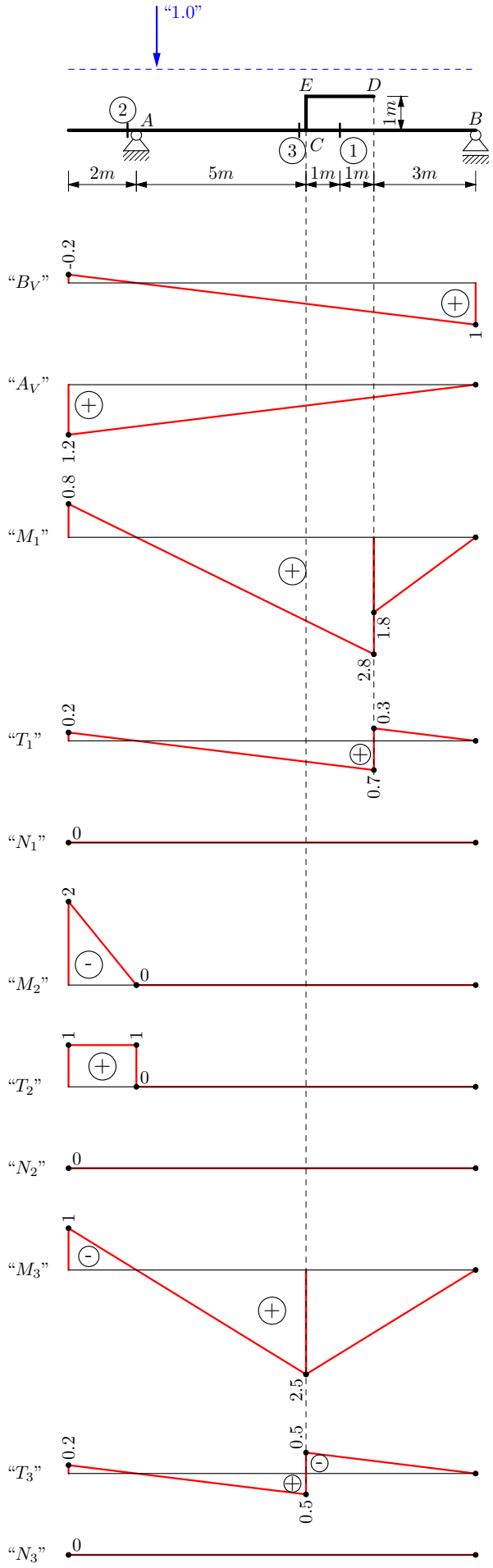
$$N_3 = 0$$

3 ↓ *B*

$$M_3 = 5 \cdot A_V = \frac{12-x}{2}$$

$$T_3 = -A_V = \frac{x-12}{10}$$

$$N_3 = 0$$



Slika 2.10

Horizontalna pokretna sila

$$\rightarrow: \quad A_H = 1.0$$

$$\overset{\curvearrowleft}{A}: \quad B_V \cdot 10 - 1.0 \cdot y = 0$$

$$\Rightarrow B_V = \frac{y}{10}$$

$$\overset{\curvearrowright}{B}: \quad A_V \cdot 10 + 1.0 \cdot y = 0$$

$$\Rightarrow A_V = -\frac{y}{10}$$

Presjek 1

$$M_1 = 4 \cdot B_V = \frac{2}{5} y$$

$$T_1 = B_V = \frac{y}{10}$$

$$N_1 = 0$$

Presjek 2

$$M_2 = 0$$

$$T_2 = 0$$

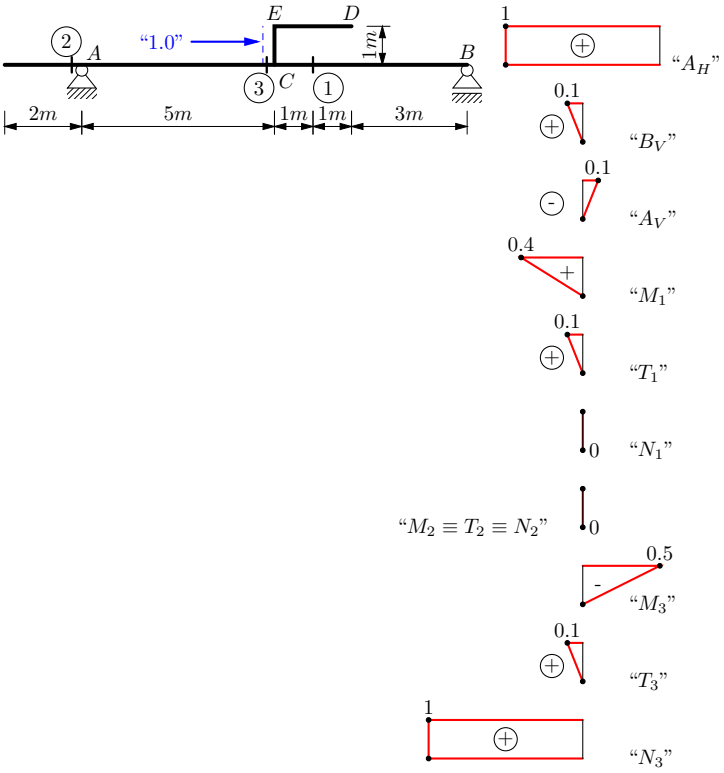
$$N_2 = 0$$

Presjek 3

$$M_3 = 5 \cdot A_V = -\frac{y}{2}$$

$$T_3 = -A_V = \frac{y}{10}$$

$$N_3 = A_H = 1.0$$



Slika 2.11

c) Provjera presječnih sila koristeći uticajne linije

Pošto vrijednost uticajne linije η_m predstavlja vrijednost uticaja (dakle momenta, poprečne ili normalne sile) u posmatranom presjeku ili reakcije na mjestu reakcije, od sile $P = 1$, vrijednost uticaja od stvarnog opterećenja ćemo dobiti množeći vrijednosti uticajne linije sa opterećenjem.

$$Z_r = \sum_i F_i \cdot \eta_i + \int q(x) \cdot \eta(x)\,\mathrm{d}x \tag{2.1}$$

gdje su

- Z_r - stvarna vrijednost uticaja (reakcija, momenat, poprečna ili normalna sila) u posmatranom presjeku (za presječne sile),
- F_i - stvarne koncentrične sile,
- η_i - vrijednosti uticajne linije ispod mjesta djelovanja koncentričnih sila F_i,
- $q(x)$ - stvarno promjenjivo opterećenje. U slučaju da je $q(x) = \text{const}$ onda može izaći ispred integrala pa je vrijednost uticaja od ravnomjerno podjeljenog opterećenja

  $$Z_r = q \int \eta(x)\,\mathrm{d}x \tag{2.2}$$

  Integral $\int \eta(x)\,\mathrm{d}x$ predstavlja površinu koju funkcija $\eta(x)$ zatvara sa x osom. Pošto su kod statički određenih nosača uticajne linije uvijek linearne funkcije, onda se površina ispod uticajne linije uvijek može proračunati bez integraljenja, već direktnim računanjem površine geometrijske figure koju zatvara uticajna linija. Na primjer prvi član u sljedećoj jednakosti kod proračuna vrijednosti momenta M_1, $\frac{0.8 \cdot 2}{2}$ predstavlja površinu trougla koji uticajna linija za "M_1" zaklapa, za vrijednosti $0 \leq x \leq 2$, koja je onda pomnožena sa vrijednosti kontinualnog opterećenja 20. Predznaci se naravno uvažavaju kako uticajnih linija tako i opterećenja.

$$\begin{aligned} M_1 &= -\frac{0.8 \cdot 2}{2} \cdot 20 + \frac{2.8 \cdot 7}{2} \cdot 20 + \frac{1.8 \cdot 3}{2} \cdot 20 + \frac{80}{3} \cdot 1.8 \\ &\quad - 60 \cdot \frac{2}{5} = 258 \\ T_1 &= 0.2 \cdot 20 + 2.45 \cdot 20 - 0.45 \cdot 20 - 80 \cdot 0.1 \\ &\quad - 60 \cdot 0.1 = 22 \\ N_1 &= 0 \end{aligned}$$

$$\begin{aligned} M_2 &= -40 \\ T_2 &= 40 \\ N_2 &= 0 \end{aligned}$$

$$\begin{aligned} M_3 &= -20 + 125 + 125 + 80 \cdot 0.5 + 30 = 300 \\ T_3 &= -4 + 25 - 25 - 80 \cdot 0.1 - 60 \cdot \frac{1}{10} = -18 \\ N_3 &= -60 \cdot 1.0 = -60 \end{aligned}$$

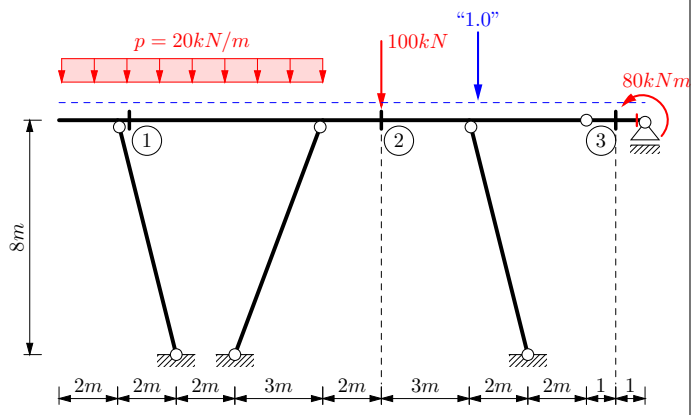
Zadatak 8 Za dati nosač i opterećenje na slici 2.17:

a) Provjeriti statičku određenost,

b) Nacrtati dijagrame momenata, transverzalnih i normalnih sila,

c) Nacrtati uticajne linije za M, T i N a za naznačeno kretanje jedinične sile,

d) Rezultate dobijene pod b) kontrolisati preko uticajnih linija dobijenih pod c).



Slika 2.17

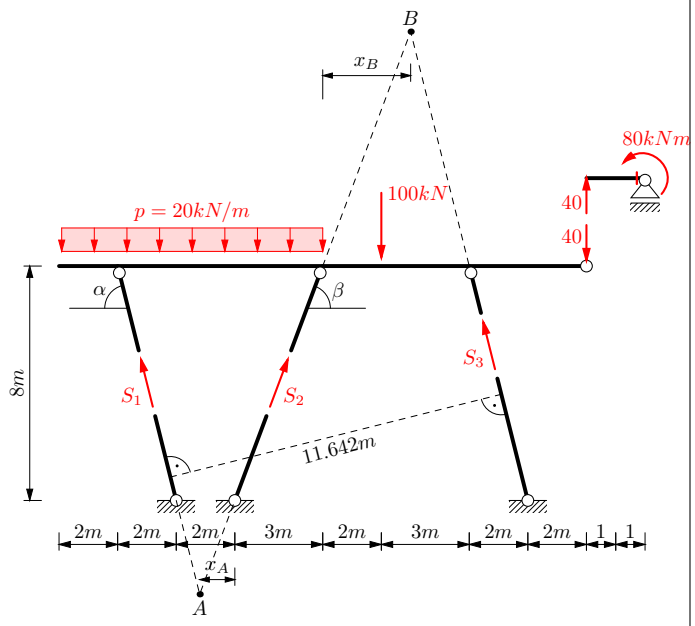
Rješenje

a) Provjera statičke određenosti

$$SSK = 2n - (s + c + r) = 2 \cdot 9 - (8 + 3 + 7) = 0$$

$\Rightarrow$ Statički određen nepokretan sistem.

b) Reakcije i presječne sile



Slika 2.18

A - presjek S_1 i S_2

$$\frac{3}{8}y_A = -\frac{2}{8}y_A + 2 \Rightarrow y_A = 3.2$$

$$x_A = \frac{3}{8} \cdot 3.2 = 1.2$$

B - presjek S_2 i S_3

$$\frac{3}{8}y_B = -\frac{2}{8}y_B + 5 \Rightarrow y_B = 8$$

$$x_B = \frac{3}{8} \cdot 8 = 3$$

$$\overset{\curvearrowleft}{A} : S_3 \cdot 11.642 - 40 \cdot 13.2 - 100 \cdot 6.2 + 180 \cdot 0.3 = 0$$

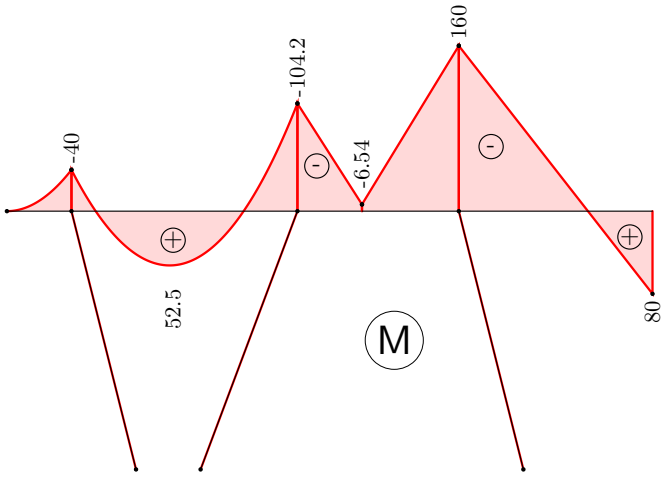
$$\Rightarrow S_3 = 93.97kN$$

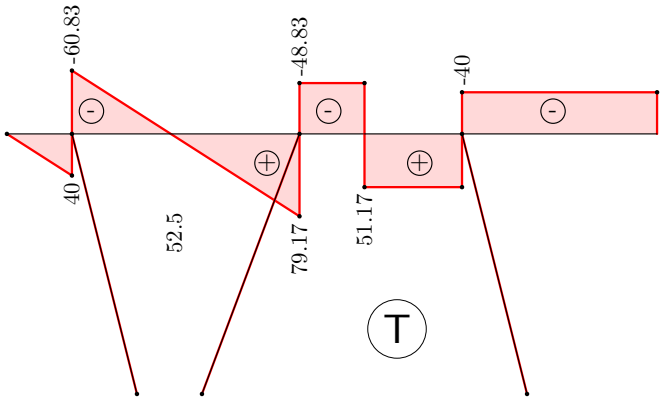
$$\overset{\curvearrowright}{B} : S_1 \cdot 11.642 - 180 \cdot 7.5 - 100 \cdot 1 + 40 \cdot 6 = 0$$

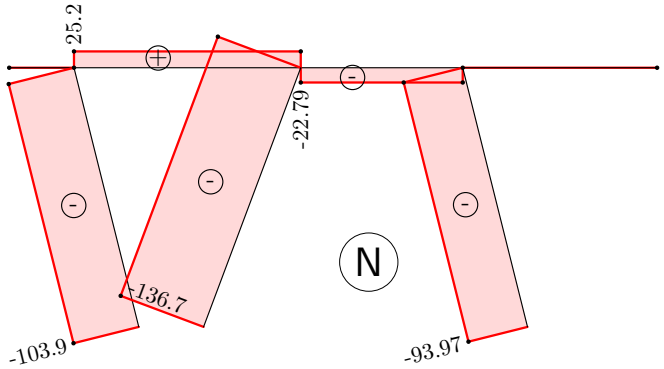
$$\Rightarrow S_1 = 103.93kN$$

$$\rightarrow : S_2 \cdot \cos\beta - S_1 \cdot \cos\alpha - S_3 \cdot \cos\alpha = 0$$

$$\Rightarrow S_2 = 136.7kN$$







c) Uticajne linije

$0 \leq x \leq 18$

$$S_1 = \frac{12 - x}{11.642}$$

$$S_2 = 0.428$$

$$S_3 = \frac{x - 4.8}{11.642}$$

$$R_C = 0$$

$18 \leq x \leq 20$

$$S_1 = (x - 20) \cdot 0.258$$

$$S_2 = -0.214x + 4.277$$

$$S_3 = (20 - x) \cdot 0.567$$

$$R_C = \frac{x - 18}{2}$$

Presjek 1

0 ↓ 1

$M_1 = -1 \cdot (2 - x)$

$T_1 = 1 - S_1 \cdot \sin\alpha = 0.083 \cdot x$

$N_1 = S_1 \cdot \cos\alpha = 0.0208(12 - x)$

1 ↓ *D*

$M_1 = 0$

$T_1 = -S_1 \cdot \sin\alpha = -0.0833 \cdot (12 - x)$

$N_1 = S_1 \cdot \cos\alpha = 0.0208(12 - x)$

D* ↓ *C

$M_1 = 0$

$T_1 = -S_1 \cdot \sin\alpha = -0.25 \cdot (20 - x)$

$N_1 = S_1 \cdot \cos\alpha = -0.0625(20 - x)$

Presjek 2

0 ↓ 2

$M_2 = 3S_3 \sin\alpha = 0.25(x - 4.8)$

$T_2 = S_3 \sin\alpha = -0.3996 + 0.0833x$

$N_2 = -S_3 \cdot \cos\alpha$

2 ↓ *D*

$M_2 = 3S_3 \sin\alpha - 1(x - 11) = 0.25(x - 4.8) - x + 11$

$T_2 = -1.399 + 0.0833x$

$N_2 = -S_3 \cdot \cos\alpha$

D* ↓ *C

$M_2 = 1.85x - 37$

$T_2 = -0.05x + 1$

$N_2 = -S_3 \cdot \cos\alpha$

Presjek 3

0 ↓ 3

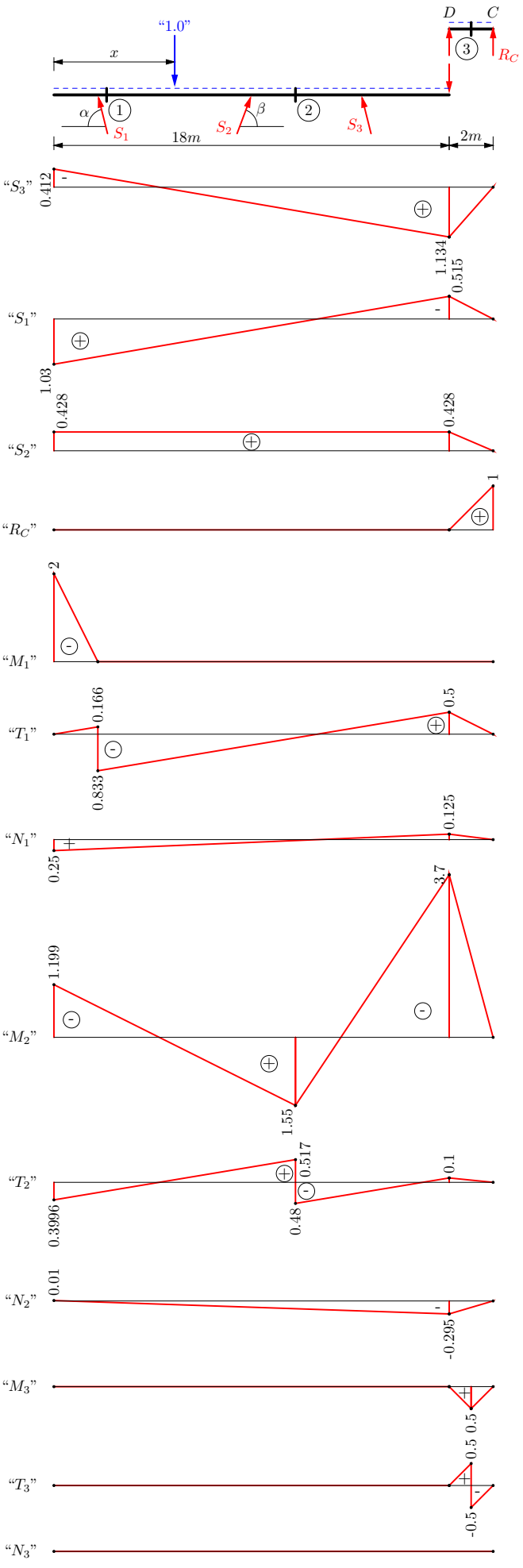
$$M_3 = R_C \cdot 1 = \frac{x - 18}{2}$$

$$T_3 = R_C = \frac{x - 18}{2}$$

3 ↓ *C*

$$M_3 = R_C - 1(x - 19) = \frac{x - 18}{2} - x + 19$$

$$T_3 = R_C - 1 = \frac{x - 18}{2} - 1$$



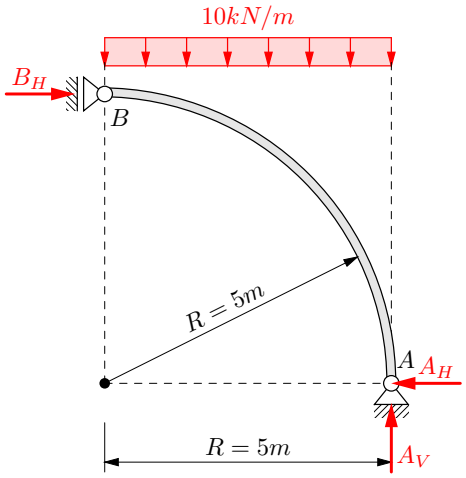
d) Provjera rezultata

$$M_1 = -40$$
$$T_1 = 3.32 - 3.79 \cdot 20 - 100 \cdot 0.0833 + 80 \cdot 0.24975 = -60.83$$
$$N_1 = 1.406 \cdot 20 + 0.02083 \cdot 100 - 80 \cdot 0.0625 = 25.2$$

$$M_2 = -20 \cdot \frac{4.8 \cdot 1.199}{2} + 20\frac{4.2 \cdot 1.05}{2} + 100 \cdot 1.5497 - 1.85 \cdot 80 = -6.5$$
$$T_2 = -19.169 - 48.33 + 4 + 14.7 = -48.8$$
$$N_2 = 4.79 - 3.67 - 12.92 - 11 = -22.8$$

$$M_3 = 0.5 \cdot 80 = 40$$
$$T_3 = -0.5 \cdot 80 = -40$$
$$N_3 = 0$$

Zadatak 11 Odrediti reakcije, presječne sile i uticajne linije za nosač na slici 2.23



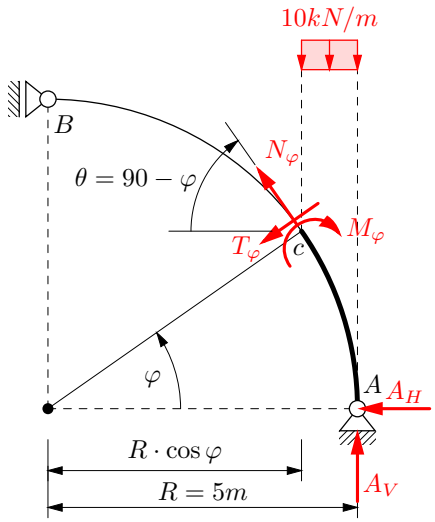
Slika 2.23

Rješenje

Reakcije

$$\uparrow: \; A_V - 10 \cdot 5.0 = 0 \qquad \Rightarrow A_V = 50kN$$
$$\curvearrowleft_A: \; B_H \cdot 5 - 10 \cdot 5 \cdot 2.5 = 0 \Rightarrow B_H = 25kN$$
$$\rightarrow: \; A_H - B_H = 0 \qquad \Rightarrow A_H = 25kN$$

Presječne sile



Slika 2.24

$$\curvearrowright_c: \; M_\varphi + A_H \cdot R \cdot \sin\varphi - A_V \cdot R \cdot (1 - \cos\varphi) + 10 \cdot \frac{R^2 \cdot (1-\cos\varphi)^2}{2} = 0$$

$$\begin{aligned} \Rightarrow \; M_\varphi &= A_V \cdot R \cdot (1-\cos\varphi) - A_H \cdot R \cdot \sin\varphi \\ &\quad - 10 \cdot \frac{R^2 \cdot (1-\cos\varphi)^2}{2} \\ &= 50 \cdot 5 \cdot (1-\cos\varphi) - 25 \cdot 5 \cdot \sin\varphi \\ &\quad - 10 \cdot \frac{5^2 \cdot (1-\cos\varphi)^2}{2} \\ &= 250 \cdot (1-\cos\varphi) - 125 \cdot \sin\varphi - 125 \cdot (1-\cos\varphi)^2 \\ &= 125 \cdot (-\sin\varphi - \cos^2\varphi + 1) \\ &= 125 \cdot (\sin^2\varphi - \sin\varphi) \end{aligned}$$

Suma horizontalnih i vertikalnih sila daje

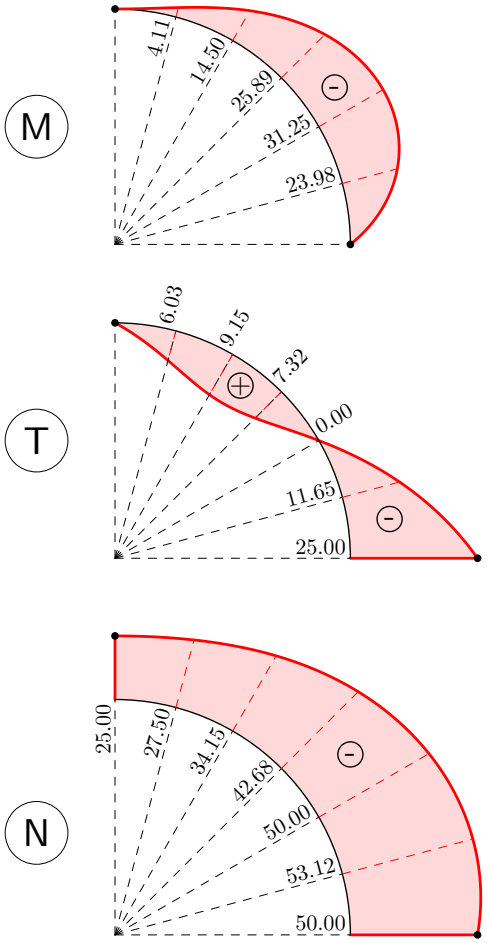
$$H_\varphi = -25$$
$$V_\varphi = 50 - 10 \cdot 5 \cdot (1 - \cos\varphi)$$

Koristeći odnose $\sin(90° - \varphi) = \cos\varphi$ i $\cos(90° - \varphi) = \sin\varphi$ dobijamo obrazce za transformaciju

$$N_\varphi = H_\varphi \cdot \sin\varphi - V_\varphi \cdot \cos\varphi$$
$$T_\varphi = H_\varphi \cdot \cos\varphi + V_\varphi \cdot \sin\varphi$$

i konačno normalnu i poprečnu silu u zavisnosti od ugla φ

$$N_\varphi = -25 \cdot \sin\varphi - 50 \cdot \cos^2\varphi$$
$$T_\varphi = -25 \cdot \cos\varphi + 50 \cdot \cos\varphi \sin\varphi$$



Slika 2.25

Zadatak 16 Za konstrukciju na slici 2.31:

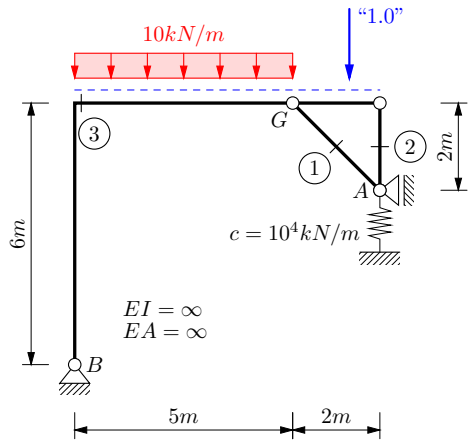
a) odrediti stepen statičke neodređenosti,

b) proračunati reakcije, presječne sile i nacrtati dijagrame presječnih sila,

c) proračunati i nacrtati uticajne linije za reakcije, momenat, poprečnu i normalnu silu u presjecima 1, 2 i 3

d) kontrolisati presječne sile u presjecima 1, 2 i 3 proračunate pod tačkom b) koristeći uticajne linije proračunate pod tačkom c),

e) proračunati pomjeranje tačke A.



Slika 2.31

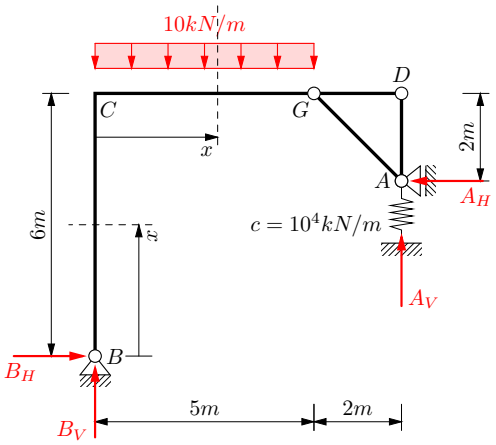
Rješenje

a) Stepen statičke neodređenosti

$SSK = 2 \cdot n - (s + c + r) = 2 \cdot 5 - (5 + 1 + 4) = 0$

$\Rightarrow$ Statički određen nepokretan sistem.

b) Reakcije i presječne sile



Slika 2.32

$\overset{D}{\curvearrowleft} G$: $A_V \cdot 2 - A_H \cdot 2 = 0 \qquad \Rightarrow A_H = A_V$

$\overset{}{\curvearrowleft} B$: $A_V \cdot 7 + A_H \cdot 4 - 10 \cdot 5 \cdot 2.5 = 0$

$A_V \cdot 7 + A_V \cdot 4 - 125 = 0 \qquad \Rightarrow A_V = A_H = 11.36kN$

$\left.\begin{array}{ll} \overset{}{\curvearrowleft} A & : B_V \cdot 7 - B_H \cdot 4 - 10 \cdot 5 \cdot 4.5 = 0 \\ \overset{L}{\curvearrowleft} G & : B_V \cdot 5 - B_H \cdot 6 - 10 \cdot 5 \cdot 2.5 = 0 \end{array}\right| \cdot 1.5 \Big\} \ominus$

$\Rightarrow B_V \cdot (7 \cdot 1.5 - 5) - 225 \cdot 1.5 + 125 = 0 \Rightarrow B_V = 38.63kN$

$B_H = (38.63 \cdot 7 - 225)/4 \Rightarrow B_H = 11.36kN$

Provjera

$\rightarrow : \; B_H - A_H = 11.36 - 11.36 = 0$

$\uparrow : \; A_V + B_V - 10 \cdot 5 = 11.36 + 38.63 - 50 = 0$

Presječne sile

Dio B-C

$0 \le x \le 6$

$M(x) = -B_H \cdot x = -11.36 \cdot x$

$T(x) = B_H = 11.36$

$N(x) = -B_V = -38.64$

x	0	6
M	0	−68.18
T	11.36	11.36
N	−38.64	−38.64

Dio C-G

$0 \le x \le 5$

$M(x) = -B_H \cdot 6 + B_V \cdot x - 10 \cdot x\frac{x}{2}$

$= -11.36 \cdot 6 + 38.63 \cdot x - 5 \cdot x^2$

$T(x) = -B_V + 10 \cdot x = -38.63 + 10 \cdot x$

$N(x) = -B_H = -11.36$

x	0	5
M	−68.18	0.00
T	−38.64	11.36
N	−11.36	−11.36

Dio G-A

$M(x) = 0$

$T(x) = 0$

$N(x) = -A_V \cdot \sin 45 - A_H \cdot \cos 45 =$

$= -11.36 \cdot \sin 45 - 11.36 \cdot \cos 45 = -16.07kN$

c) Uticajne linije $C \downarrow G$

$\overset{D}{\curvearrowleft} G$: $A_V \cdot 2 - A_H \cdot 2 = 0 \qquad \Rightarrow A_H = A_V$

$\overset{}{\curvearrowleft} B$: $A_V \cdot 7 + A_H \cdot 4 - 1 \cdot x = 0$

$A_V \cdot 7 + A_V \cdot 4 - x = 0 \qquad \Rightarrow A_V = A_H = \frac{x}{11}$

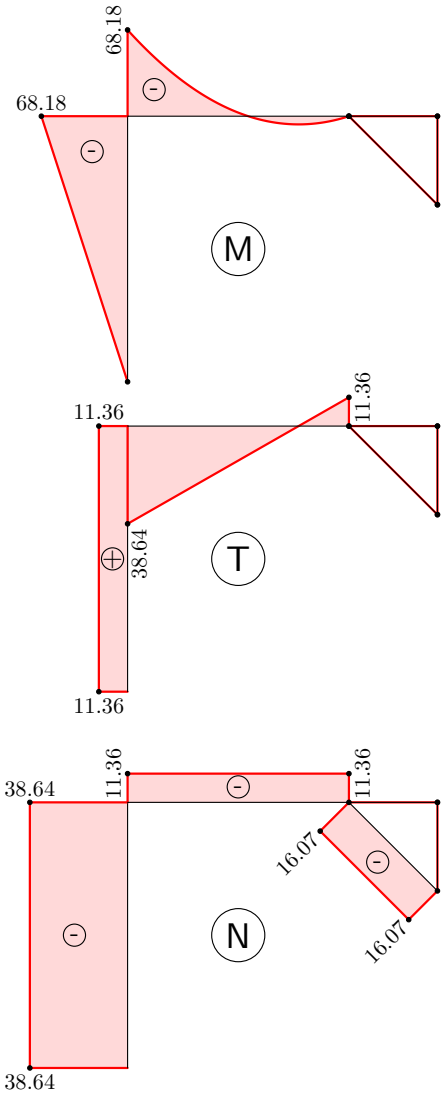
$\rightarrow : \; B_H = A_H = \frac{x}{11}$

$\overset{L}{\curvearrowleft} G$: $B_V \cdot 5 - B_H \cdot 6 - 1 \cdot (5 - x) = 0$

$B_V \cdot 5 - \frac{x}{11} \cdot 6 - 5 + x = 0$

$B_V = -\frac{1}{11}x + 1$

x	0	5
A_V	0	$^5/_{11}$
A_H	0	$^5/_{11}$
B_V	1	$^6/_{11}$
B_H	0	$^5/_{11}$



Slika 2.33

G* ↓ *D

$\overset{\curvearrowleft D}{G}: \; A_V \cdot 2 - A_H \cdot 2 - 1 \cdot (x-5) = 0$

$\overset{\curvearrowleft}{B}: \; A_V \cdot 7 + A_H \cdot 4 - 1 \cdot x = 0$

$(2 \cdot 2 + 7) A_V - 2 \cdot (x-5) - x = 0$

$\Rightarrow A_V = \dfrac{3x-10}{11}$

$A_H = \dfrac{-A_V \cdot 7 + x}{4} = \dfrac{-\frac{3x-10}{11} \cdot 7 + x}{4}$

$\Rightarrow A_H = \dfrac{70-10x}{44}$

$\rightarrow: \; B_H = A_H = \dfrac{70-10x}{44}$

$\overset{\curvearrowright L}{G}: \; B_V \cdot 5 - B_H \cdot 6 = 0$

$: \; B_V = \dfrac{6}{5} B_H = \dfrac{6(70-10x)}{220}$

x	5	7
A_V	$^5/_{11}$	1
A_H	$^5/_{11}$	0
B_V	$^6/_{11}$	0
B_H	$^5/_{11}$	0

Presjek 2

C* ↓ *G

$M_2 = 0$

$T_2 = 0$

$N_2 = 0$

G* ↓ *D

$M_2 = 0$

$T_2 = 0$

$N_2 = \dfrac{x-5}{2}$

Presjek 1

C* ↓ *G

$M_1 = 0$

$T_1 = 0$

$N_1 = -A_V \cdot \sin 45 - A_H \cdot \cos 45 =$

$= -\dfrac{\sqrt{2}}{2}\dfrac{x}{11} - \dfrac{\sqrt{2}}{2}\dfrac{x}{11} = -\dfrac{\sqrt{2}}{11}x$

x	0	5
N_1	0	−0.643

G* ↓ *D

$M_1 = 0$

$T_1 = 0$

$N_1 = -\dfrac{A_H}{\cos 45} = -\dfrac{70-10x}{44}\dfrac{1}{\frac{\sqrt{2}}{2}} = -\dfrac{70-10x}{44}\dfrac{2}{\sqrt{2}} \cdot \dfrac{\sqrt{2}}{\sqrt{2}} =$

$= -\dfrac{(70-10x)\sqrt{2}}{44}$

x	5	7
N_1	−0.643	0

Presjek 3

C* ↓ *G

$M_3 = -B_H \cdot 6 = -\dfrac{6}{11}x$

$T_3 = -B_V = \dfrac{1}{11}x - 1$

$N_3 = -B_H = -\dfrac{1}{11}x$

x	0	5
M_3	0	−2.727
T_3	−1	−0.545
N_3	0	−0.455

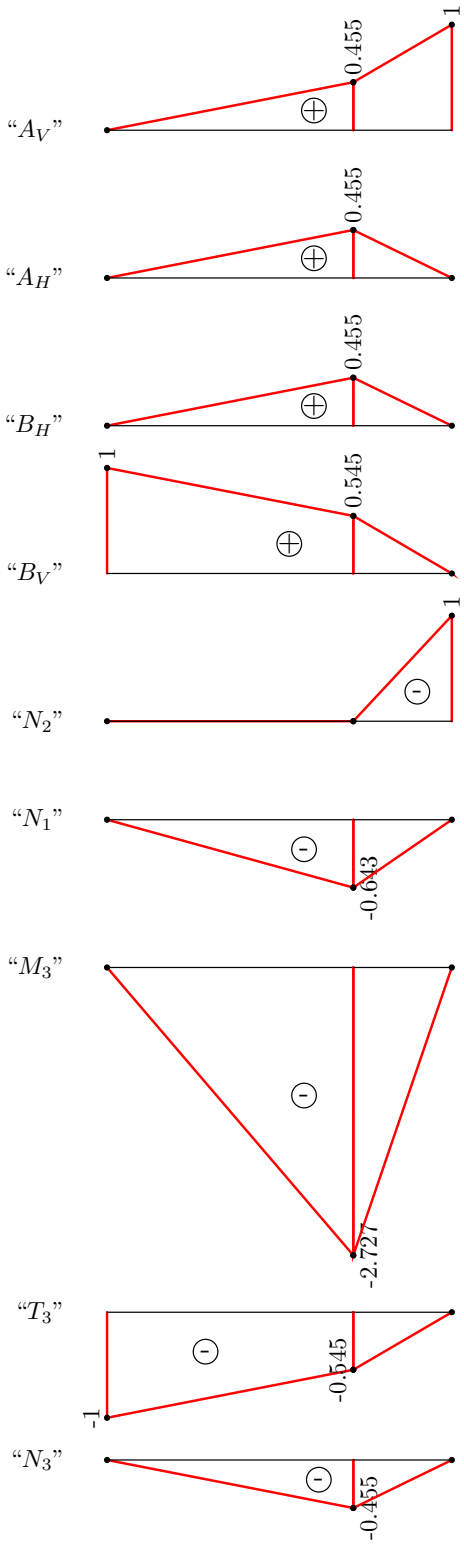
G* ↓ *D

$M_3 = -B_H \cdot 6 = -6\dfrac{70-10x}{44}$

$T_3 = -B_V = -\dfrac{6(70-10x)}{220}$

$N_3 = -B_H = -\dfrac{70-10x}{44}$

x	5	7
M_3	−2.727	0
T_3	−0.545	0
N_3	−0.455	0



Slika 2.34

d) Provjera presječnih sila koristeći uticajne linije

$$N_1 = \frac{-0.643 \cdot 5}{2} \cdot 10 = -16.08kN$$

$$M_3 = \frac{-2.727 \cdot 5}{2} \cdot 10 = -68.18kN$$

$$T_3 = \frac{-1 + (-0.545)}{2} \cdot 5 \cdot 10 = -38.63kN$$

$$N_3 = \frac{-0.455 \cdot 5}{2} \cdot 10 = -11.36kN$$

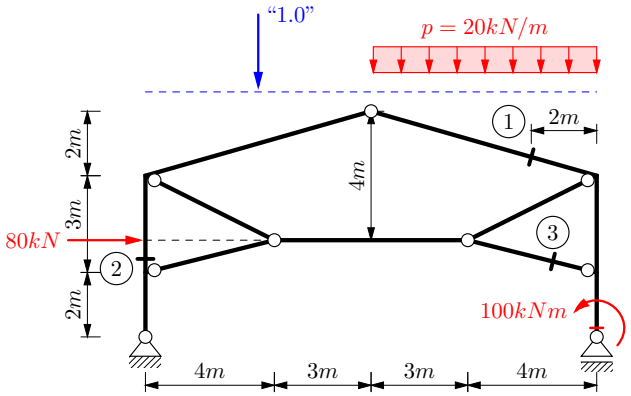
e) Pomjeranje tačke A

$$\delta_A = \frac{A_V}{c} = \frac{11.36kN}{10^4 kN/m} = 0.001136m$$

Zadatak 18 Za konstrukciju na slici 2.36 proračunati:

a) Dijagrame presječnih sila,

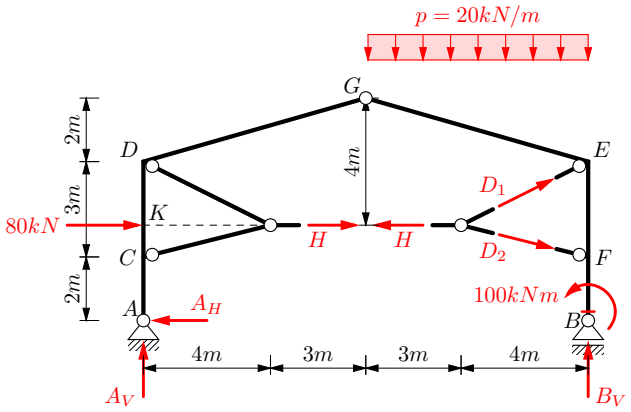
b) Uticajne linije za presječne sile u datim presjecima.



Slika 2.36

Rješenje

Određivanje reakcija



Slika 2.37

$\overset{\curvearrowleft}{A}: 100 + 14 \cdot B_V - 20 \cdot 7 \cdot 10.5 - 80 \cdot 3 = 0$

$\Rightarrow B_V = 115kN$

$\overset{\curvearrowleft}{B}: 100 + 140 \cdot 3.5 - 80 \cdot 3 - A_V \cdot 14 = 0$

$\Rightarrow A_V = 25kN$

$\rightarrow: 80 - A_H = 0$

$\Rightarrow A_H = 80kN$

$\overset{\curvearrowleft}{G_L}: 80 \cdot 4 + H \cdot 4 - A_H \cdot 7 - A_V \cdot 7 = 0$

$\Rightarrow H = 103.75kN$

Presječne sile

$M_C = 160$

$M_K = 170.83$

$M_D = 32.49$

$M_B = 100$

$M_E = -107.5$

$_{max}M_{G-E} = 74.66$

$D_1 = 38.67$

$D_2 = 71.3$

$T_A = -80$

$T_C^D = -10.83$

$T_K^D = 69.17$

$T_D^L = 69.17$

$T_D^D = 4.46$

$T_F^L = -69.17$

$T_E^D = -69.17$

$T_E^L = 82.1$

$T_G^D = -52.5$

$T_G^L = 4.46$

$N_A = -25$

$N_C^D = -42.29$

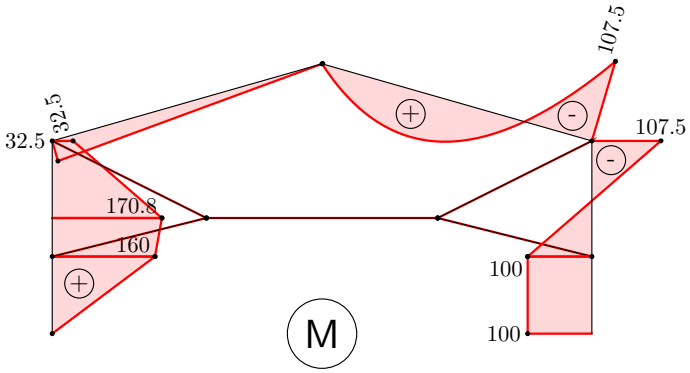
$N_D^D = -106.63$

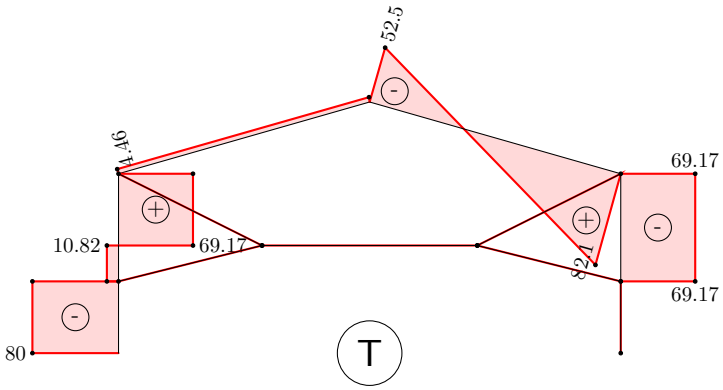
$N_B = -115$

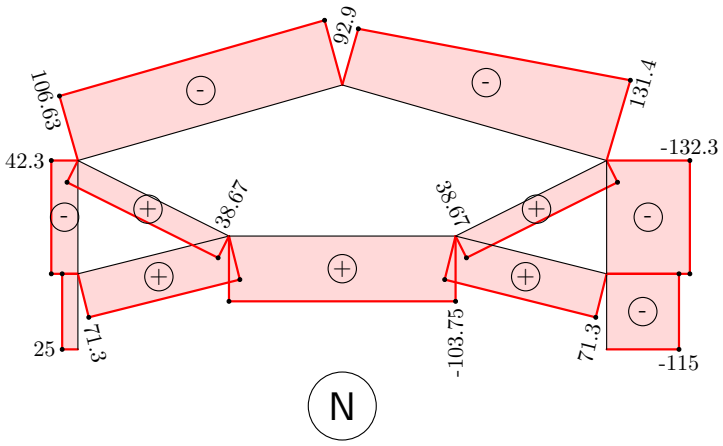
$N_F^L = -132.3$

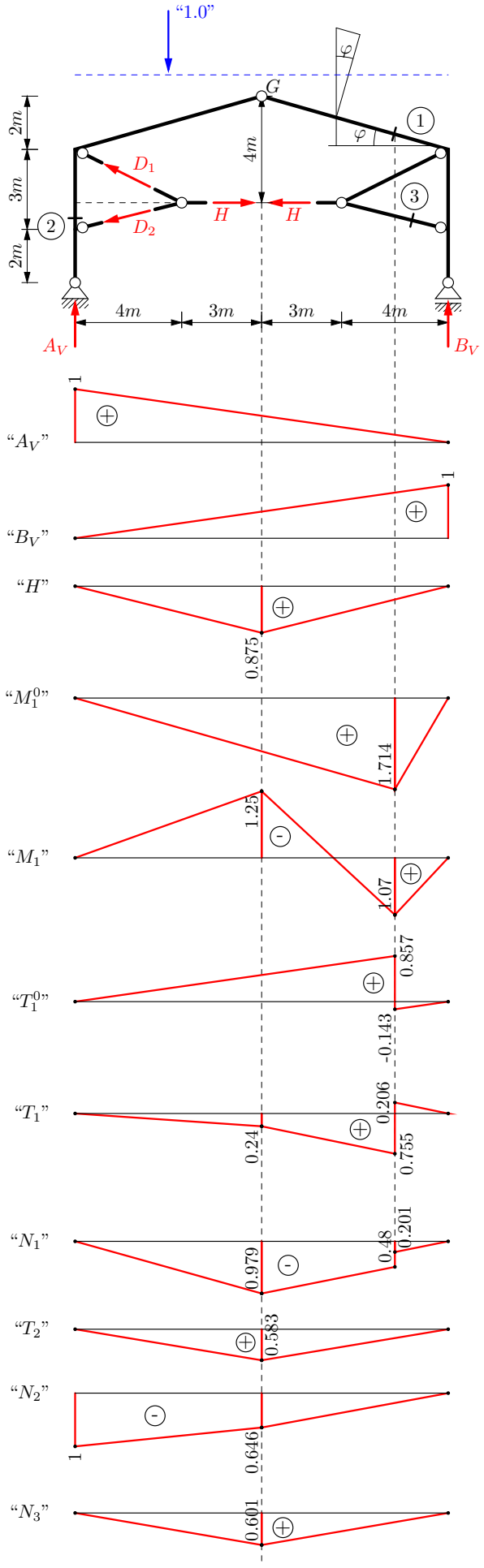
$N_E^L = -131.4$

$N_G^D = -92.94$









Presjek 1

$$H = \frac{M_G^0}{4}$$

$$M_1 = M_1^0 - H \cdot 2.571$$

$$T_1 = T_0 \cdot \cos\varphi - H \cdot \sin\varphi$$

$$N_1 = -T_0 \cdot \sin\varphi - H \cdot \cos\varphi$$

$$\cos\varphi = 0.9615$$

$$\sin\varphi = 0.2747$$

Presjek 2

$$M_2 = 0$$

$$T_2 = 0.666 \cdot H$$

$$D_2 = 0.687 \cdot H$$

$$N_2 = -A - D_2^V$$

$$= -A - 0.1667 \cdot H$$

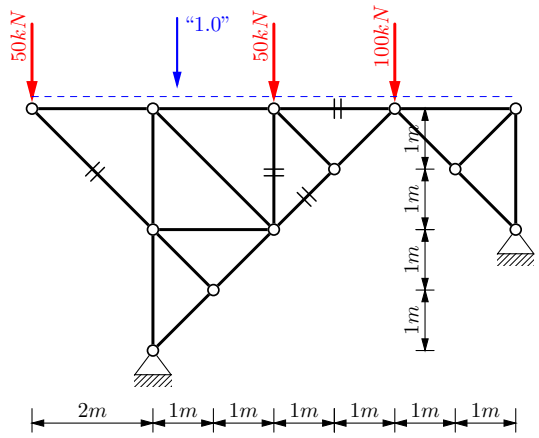
Presjek 3

$$M_3 = 0$$

$$T_3 = 0$$

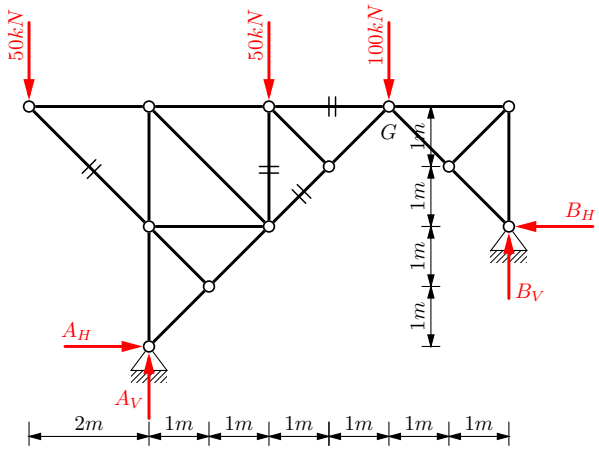
$$N_3 = D_2 = 0.687 \cdot H$$

Zadatak 19 Odrediti reakcije, presječne sile i uticajne linije u označenim štapovima za rešetku prikazanu na slici 2.38. Presječne sile kontrolisati integracijom uticajnh linija.



Slika 2.38

Rješenje



Slika 2.39

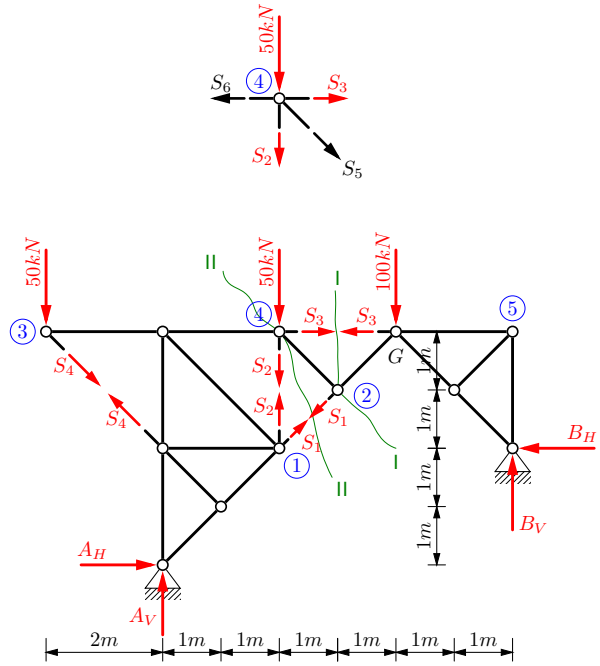
Reakcije

$$\overset{\curvearrowleft}{A}: B_V \cdot 6 + B_H \cdot 2 - 100 \cdot 4 - 50 \cdot 2 + 50 \cdot 2 = 0$$

$$\overset{\curvearrowleft}{G_D}: B_V \cdot 2 - B_H \cdot 2 = 0$$
$$\Rightarrow B_V = B_H$$
$$: B_V \cdot 8 = 400$$
$$\Rightarrow B_V = B_H = 50$$
$$\rightarrow A_H - B_H = 0$$
$$\Rightarrow A_H = 50$$
$$\overset{\curvearrowright}{B}: A_V \cdot 6 - A_H \cdot 2 - 100 \cdot 2 - 50 \cdot 4 - 50 \cdot 8 = 0$$
$$\Rightarrow A_V = 150kN$$

Presječne sile

$$\overset{\curvearrowright}{2}_L: S_3 \cdot 1 - 50 \cdot 1 - 50 \cdot 5 - A_H \cdot 3 + A_V \cdot 3 = 0$$
$$\Rightarrow S_3 = 0$$
$$\overset{\curvearrowleft}{4}_L: S_1 \cdot \sqrt{2} + 50 \cdot 4 + A_H \cdot 4 - A_V \cdot 2 = 0$$
$$\Rightarrow S_1 = -70.71kN$$



Slika 2.40

Čvor 2

$$\Rightarrow S_5 = 0$$

Čvor 4

$$\Rightarrow S_6 = 0$$
$$\uparrow: S_2 = -50kN$$

Čvor 3

$$\uparrow: S_4 \cdot \cos(45^\circ) + 50 = 0$$
$$\Rightarrow S_4 = -70.71kN$$

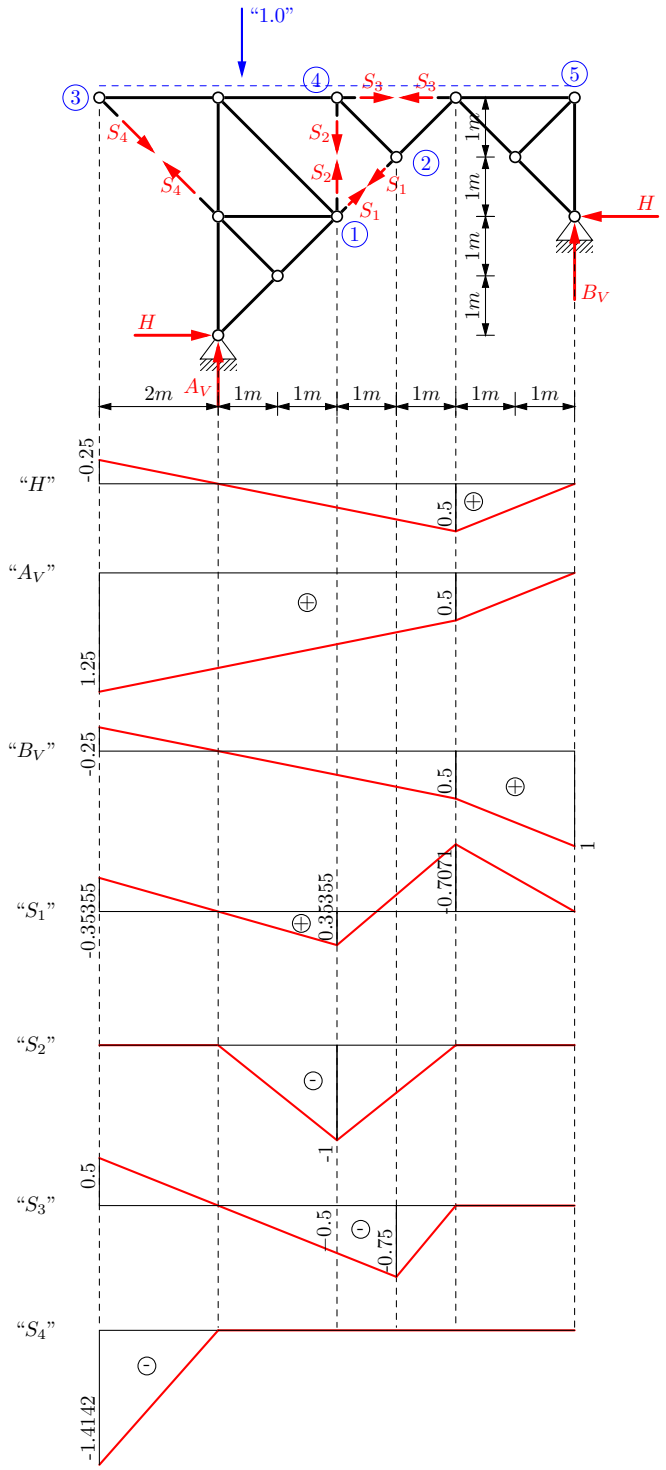
Uticajne linije

③ ↓ *G*

$$\overset{\curvearrowleft}{G_D}: B_V \cdot 2 - H \cdot 2 = 0$$
$$\Rightarrow H = B_V$$
$$\overset{\curvearrowleft}{A}: B_V \cdot 6 + H \cdot 2 - 1 \cdot x = 0$$
$$\Rightarrow H = B_V = \frac{x}{8}$$

G **↓ ⑤**

$$\overset{\curvearrowleft}{G_L}: A_V \cdot 4 - H \cdot 4 = 0$$
$$\Rightarrow H = A_V$$
$$\overset{\curvearrowright}{B}: A_V \cdot 6 - H \cdot 2 - (6 - x) \cdot 1 = 0$$
$$\Rightarrow H = A_V = \frac{6 - x}{4}$$



Slika 2.41

Sila S_1

③ ↓ ④

$$\overset{\curvearrowleft}{4}_D : S_1 \cdot \sqrt{2} + H \cdot 2 - B_V \cdot 4 = 0$$
$$\Rightarrow S_1 = \sqrt{2}\,(2 \cdot B_V - H)$$

④ ↓ ⑤

$$\overset{\curvearrowleft}{4}_L : S_1 \cdot \sqrt{2} + H \cdot 4 - A_V \cdot 2 = 0$$
$$\Rightarrow S_1 = \sqrt{2}\,(A_V - 2 \cdot H)$$

Sila S_2

③ ↓ ④

$$\uparrow : -S_2 - S_1 \cdot \frac{\sqrt{2}}{2} + B_V = 0$$
$$\Rightarrow S_2 = B_V - S_1 \cdot \frac{\sqrt{2}}{2}$$

④ ↓ ⑤

$$\uparrow : S_2 + S_1 \cdot \frac{\sqrt{2}}{2} + A_V = 0$$
$$\Rightarrow S_2 = -A_V - S_1 \cdot \frac{\sqrt{2}}{2}$$

Sila S_3

③ ↓ ②

$$\overset{\curvearrowleft}{2}_D : S_3 \cdot 1 + B_V \cdot 3 - H \cdot 1 = 0$$
$$\Rightarrow S_3 = H - 3 \cdot B_V$$

② ↓ ⑤

$$\overset{\curvearrowleft}{2}_L : S_3 \cdot 1 + A_V \cdot 3 - H \cdot 3 = 0$$
$$\Rightarrow S_3 = 3 \cdot H - 3 \cdot A_V$$

Sila S_4

$0 \leq x \leq 2$

$$\uparrow : S_4 \frac{\sqrt{2}}{2} - 1.0 \cdot \frac{2-x}{2} = 0$$
$$\Rightarrow S_4 = \frac{\sqrt{2}}{2}(2-x)$$

Integracija uticajnih linija

$S_1 = -0.35355 \cdot 50 + 0.35355 \cdot 50 - 0.7071 \cdot 100 = -70.71kN$
$S_2 = -1 \cdot 50 = -50.0kN$
$S_3 = 0.5 \cdot 50 - 0.5 \cdot 50 + 0 \cdot 100 = 0.0kN$
$S_4 = -1.4142 \cdot 50 = -70.71kN$

Uticajna linija za štap S_{11}

Za položaj jedinične sile iznad tačke 3 proračunavamo reakcije

$$B_V = -0.25$$
$$A_V = 1.25$$
$$H = -0.25$$

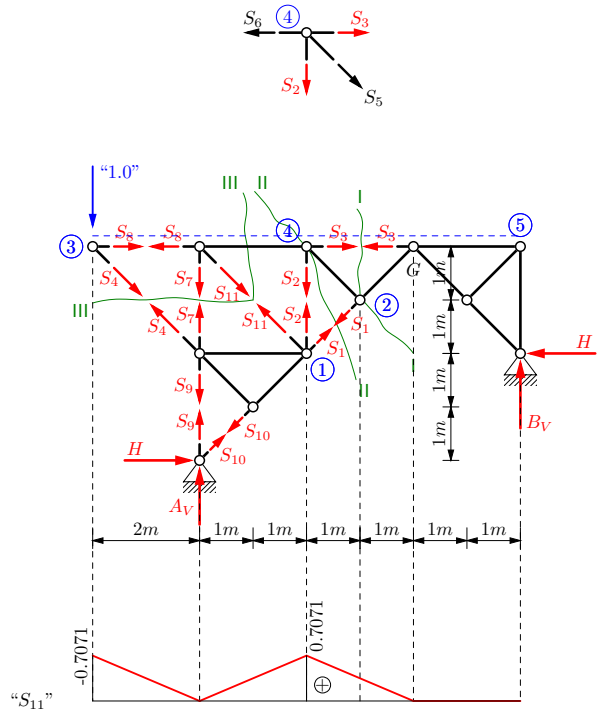
i sile u štapovima $S_4, S_7, S_8, S_9, S_{10}$

$$S_4 = -1.414$$
$$S_7 = -0.5$$
$$S_8 = 1.0$$
$$S_9 = -1.50$$
$$S_{10} = 0.353$$

Iz uslova ravnoteže vertikalnih sila u presjeku III − III dobijamo

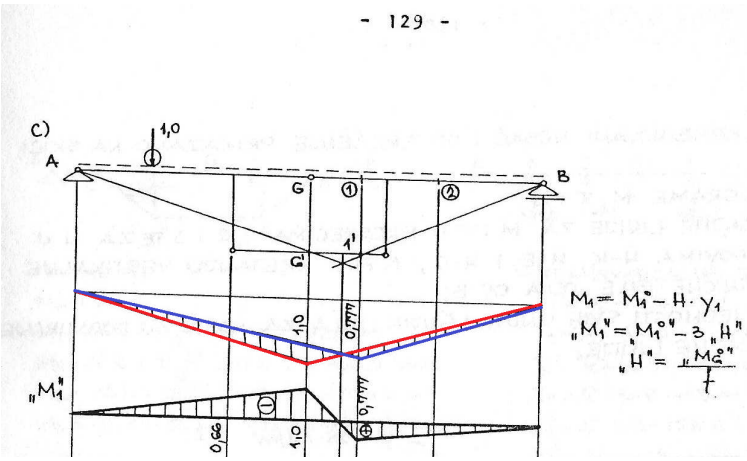
$$S_{11} \cdot \sin(45^\circ) + S_7 + S_4 \cdot \sin(45^\circ) = 0$$
$$\Rightarrow S_{11} = 0.707$$

Kad je jedinična sila iznad oslonca A sile u svim štapovima su jednake nuli osim u vertkalnim štapovima iznad oslonca A. Kad se jednnična sila nalazi iznad tačke 4 proračunavamo kao za položaj iznad tačke 3.



Slika 2.42

Zadatak 1 Studentsko pitanje je bilo kako je vrijednost uticajne linije "M_1" ispod zgloba G 1.0? Pošto je nešto slično moguće da dođe i na ispitu reći ćemo i za ostale studente da se radi o zadatku datom u dokumentu Vjezbe6-Z1-Z2-Z3 - prvi zadatak u ovom dokumentu, pod oznakom 42



Slika 2.52: Plavom bojom je označeno M_1^0 ; crvenom bojom je označeno $H \cdot y$

Ponovićemo formule date na slici desno, dakle momenat

$$M_1 = M_1^0 - H \cdot y$$

$$H = \frac{M_G^0}{f}$$

gdje je M_1^0 momenat "kao na prostoj gredi" dakle momenat od vertikalnog opterećenja koje djeluje na neku zamišljenu prostu gredu od oslonca A do B. (u ovom zadatku f i y označavaju istu vrijednost jer je greda paralelna sa zategom)

Proračunajmo vrijednost M_1^0 ispod zgloba G.

Vrijednost M_1^0 ispod presjeka $\boxed{1}$ je (dakle maksimalna vrijednost)

$${}_{pr1}M_1^0 = \frac{l_1 l_2}{l_1 + l_2} = \frac{11 \cdot 7}{18} = \frac{77}{18}$$

Vrijednost M_1^0 ispod zgloba G

$${}_{G}M_1^0 = \frac{\frac{77}{18}}{11} \cdot 9 = \frac{77}{22} = \frac{7}{2} = 3.5$$

Proračunajmo sad vrijednost H ispod zgloba G

Vrijednost M_G^0 ispod zgloba G je ($l_1 = 9, l_2 = 9$)

$$M_G^0 = \frac{l_1 l_2}{l_1 + l_2} = \frac{9 \cdot 9}{9 + 9} = \frac{81}{18} = \frac{9}{2} = 4.5$$

sa $f = 3$ dobijamo

$${}_{G}H = \frac{4.5}{3}$$

Konačno M_1 ispod zgloba G

$${}_{G}M_1 = {}_{G}M_1^0 - 3 \cdot {}_{G}H = 3.5 - 3 \cdot \frac{4.5}{3} = -1$$

Kad smo već ovdje proračunajmo i vrijednost M_1 ispod presjeka $\boxed{1}$ koja je na slici data sa 0.777

Dakle, vrijednost M_1^0 ispod presjeka $\boxed{1}$ je (već proračunata)

$${}_{pr1}M_1^0 = \frac{77}{18}$$

Proračunajmo vrijednost H ispod presjeka $\boxed{1}$

M_G^0 ispod G (već proračunato)

$M_G^0 = 4.5$

M_G^0 ispod $\boxed{1}$

$${}_{pr1}M_G^0 = \frac{4.5}{9} \cdot 7 = 3.5$$

pa je

$${}_{pr1}H = \frac{3.5}{3}$$

Konačno M_1 ispod presjeka $\boxed{1}$

$${}_{pr1}M_1 = {}_{pr1}M_1^0 - 3 \cdot {}_{pr1}H = \frac{77}{18} - 3 \cdot \frac{3.5}{3} = 0.7778$$

(na oznake uticajnih linija staviti navodnike: "M_1")

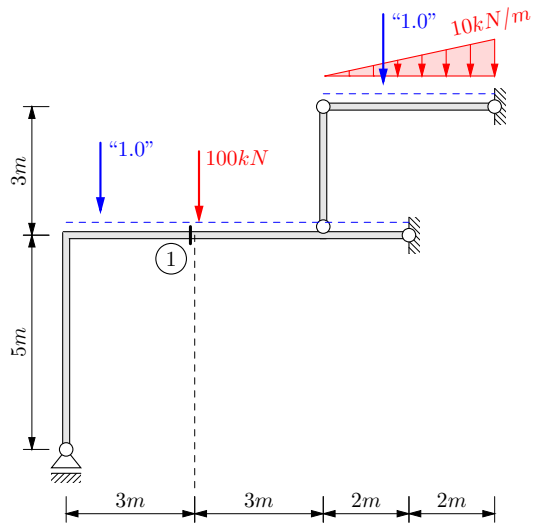
Zadatak 2 Za konstrukciju na slici 2.53

a) odrediti stepen statičke neodređenosti,

b) proračunati reakcije, presječne sile i nacrtati dijagrame presječnih sila,

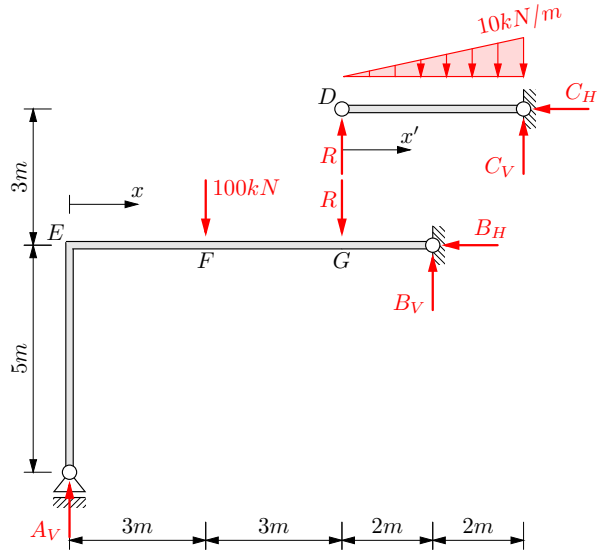
c) proračunati i nacrtati uticajne linije za reakcije, momenat, poprečnu i normalnu silu u presjeku 1,

d) kontrolisati presječne sile u presjeku 1 proračunate pod tačkom b) koristeći uticajne linije proračunate pod tačkom c).



Slika 2.53

Rješenje



Slika 2.54

Reakcije

$\curvearrowright C$: $R \cdot 4 - \frac{10 \cdot 4}{2} \cdot \frac{1}{3} \cdot 4 = 0$

$\Rightarrow R = 6.67kN$

$\curvearrowright D$: $C_V \cdot 4 - \frac{10 \cdot 4}{2} \cdot \frac{2}{3} \cdot 4 = 0$

$\Rightarrow C_V = 13.33kN$

$\curvearrowright B$: $A_V \cdot 8 - 100 \cdot 5 - R \cdot 2 = 0$

$\Rightarrow A_V = 64.17kN$

$\curvearrowright E$: $B_V \cdot 8 - 100 \cdot 3 - R \cdot 6 = 0$

$\Rightarrow B_V = 42.50kN$

$\rightarrow \Rightarrow B_H = 0$

Presječne sile

Dio D-C

$0 \leq x' \leq 4$

$$M(x') = R \cdot x' - \frac{\left(\frac{10}{4} \cdot x'\right) \cdot x'}{2} \cdot \frac{x'}{3} = 6.67 \cdot x' - 0.417 \cdot x'^3$$

$$T(x') = -R + \frac{\left(\frac{10}{4} \cdot x'\right) \cdot x'}{2} = -6.67 + 1.25 \cdot x'^2$$

$N(x') = 0$

x'	0	4
M	0	0
T	−6.67	13.33
N	0	0

Dio E-F

$0 \leq x \leq 3$

$M(x) = A_V \cdot x = 64.17 \cdot x$

$T(x) = -A_V = -64.17$

$N(x) = 0$

x	0	3
M	0	192.51
T	−64.17	−64.17
N	0	0

Dio F - G

$3 \leq x \leq 6$

$M(x) = A_V \cdot x - 100 \cdot (x - 3) = -35.83 \cdot x + 300$

$T(x) = -A_V + 100 = 35.83$

$N(x) = 0$

x	3	6
M	192.51	85.02
T	35.83	35.83
N	0	0

Dio G - B

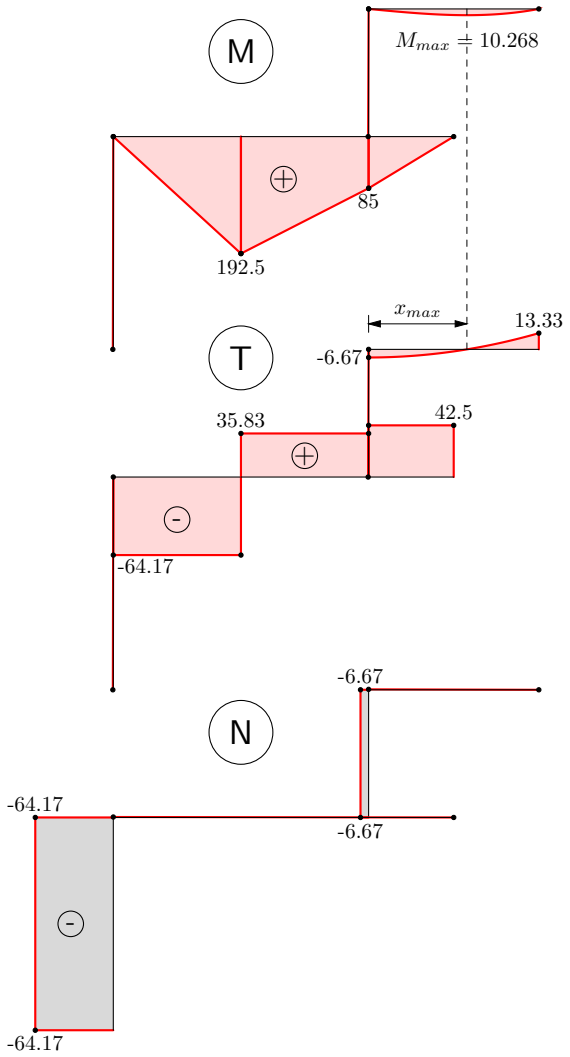
$6 \leq x \leq 8$

$M(x) = A_V \cdot x - 100 \cdot (x - 3) - R \cdot (x - 6) = -42.5 \cdot x + 340.02$

$T(x) = -A_V + 100 + 6.67 = 42.5$

$N(x) = 0$

x	6	8
M	85.02	0
T	42.5	42.5
N	0	0



Slika 2.55

Uticajne linije

Vertikalna pokretna sila "1" na gredi D-C

$\overset{\curvearrowright}{C}: \quad R \cdot 4 - 1.0 \cdot (4 - x') = 0$

$$\Rightarrow R = \frac{4 - x'}{4}$$

$\overset{\curvearrowright}{B}: \quad A_V \cdot 8 - R \cdot 2 = 0$

$$\Rightarrow A_V = \frac{4 - x'}{4} \cdot \frac{2}{8} = \frac{4 - x'}{16}$$

$\overset{\curvearrowleft}{E}: \quad B_V \cdot 8 - R \cdot 6 = 0$

$$\Rightarrow B_V = \frac{4 - x'}{4} \cdot \frac{6}{8} = \frac{(4 - x') \cdot 3}{16}$$

Presjek 1

D* ↓ *C

$$M_1 = A_V \cdot 3 = \frac{(4 - x') \cdot 3}{16}$$

$$T_1 = -A_V = -\frac{4 - x'}{16}$$

$$N_1 = 0$$

Vertikalna pokretna sila "1" na gredi E-B

$\overset{\curvearrowleft}{E}: \quad B_V \cdot 8 - 1.0 \cdot x = 0$

$$\Rightarrow B_V = \frac{x}{8}$$

$\overset{\curvearrowright}{B}: \quad A_V \cdot 8 - 1.0 \cdot (8 - x) = 0$

$$\Rightarrow A_V = \frac{8 - x}{8}$$

E* ↓ *F

$$M_1 = 5 \cdot B_V = 5 \cdot \frac{x}{8}$$

$$T_1 = B_V = \frac{x}{8}$$

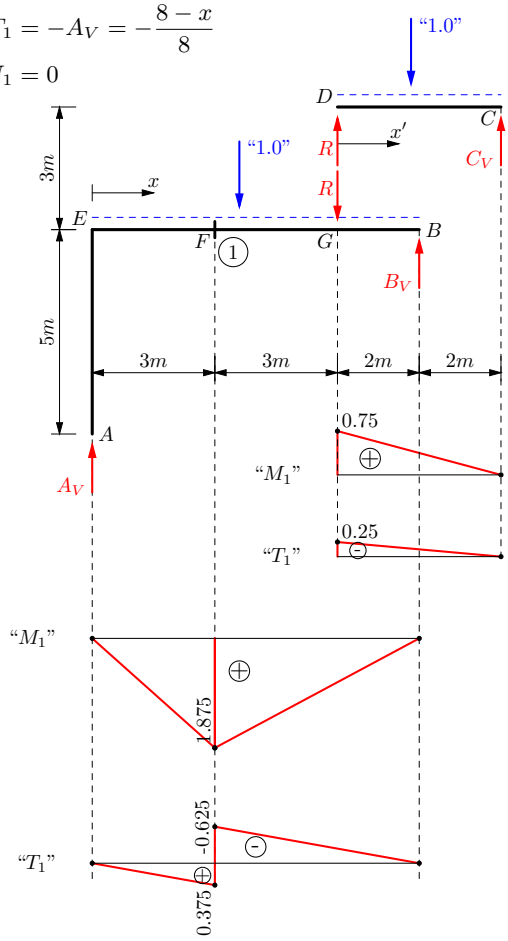
$$N_1 = 0$$

F* ↓ *B

$$M_1 = 3 \cdot A_V = 3 \cdot \frac{8 - x}{8}$$

$$T_1 = -A_V = -\frac{8 - x}{8}$$

$$N_1 = 0$$



Slika 2.56

Provjera presječnih sila koristeći uticajne linije

$$M_1 = 100 \cdot 1.875 + \frac{10 \cdot 4}{2} \cdot 0.75 \cdot \frac{1}{3} = 192.5$$

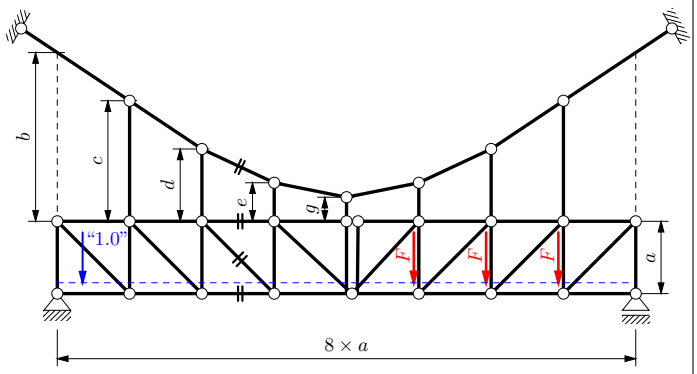
$$T_1 = -0.625 \cdot 100 - \frac{10 \cdot 4}{2} \cdot 0.25 \cdot \frac{1}{3} = -64.167$$

$$N_1 = 0$$

gdje je poprečna sila u presjeku 1 proračunata lijevo od mjesta djelovanja koncentrične sile, jer je tako dato zadatkom. U slučaju da je presjek 1 desno od sile imali bismo poprečnu silu

$$T_1 = 0.375 \cdot 100 - \frac{10 \cdot 4}{2} \cdot 0.25 \cdot \frac{1}{3} = 35.83$$

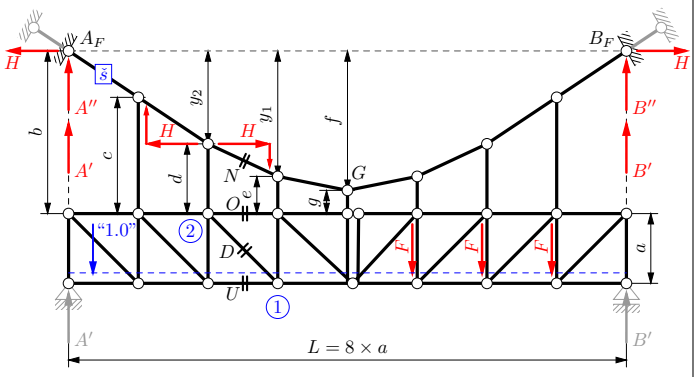
Zadatak 3 Proračunati reakcije, presječne sile i uticajne linije u naznačenim presjecima.



Slika 2.57

$a = 2.2m, \quad b = 2.33a, \quad c = 1.67a, \quad d = a, \quad e = 0.53a,$
$g = 0.33a, \quad F = 80kN$

Rješenje



Slika 2.58

Postavićemo fiktivne oslonce A_F i B_F. Pošto je fiktivni oslonac postavljen na pravac zglobno vezanog štapa, sila u štapu se ne mijenja a time ni reakcija na kraju štapa.

Ukupne vertikalne reakcije su

$$A_U = A' + A''$$
$$B_U = B' + B''$$

sa kojima možemo postaviti sljedeće uslove ravnoteže

$$\overset{\curvearrowleft}{A_F}: \; B_U \cdot L - M_A = 0 \Rightarrow B_U = \frac{M_A}{L}$$
$$\overset{\curvearrowright}{B_F}: \; A_U \cdot L - M_B = 0 \Rightarrow A_U = \frac{M_B}{L}$$

gdje je M_A ukupni momenat od opterećenja oko tačke A. Dakle, suma vertikalnih komponenata reakcija jednaka je reakciji proste grede istog raspona.

$$B_U = \frac{80 \cdot (5+6+7) \cdot 2.2}{8 \cdot 2.2} = 180kN$$
$$A_U = \frac{80 \cdot (1+2+3) \cdot 2.2}{8 \cdot 2.2} = 60kN$$

Iz uslova ravnoteže momenata oko zgloba G, sa bilo koje strane, dobijamo veličinu horizontalne komponente reakcije H

$$H = \frac{M_G^0}{f} = \frac{60 \cdot 4 \cdot 2.2}{5.126 - 0.726} = 120kN$$

gdje je M_G^0 momenat oko zgloba G "kao na prostoj gredi", dakle momenat od vertikalnog opterećenja koje djeluje na neku zamišljenu prostu gredu od oslonca A do B i od ukupne reakcije A_U ili B_U

Nagib štapa označenog na slici sa š se može proračunati pa je

$$\tan\alpha = \frac{b-c}{a} = \frac{5.126 - 3.674}{2.2} = 0.66$$

pa se može proračunati i vrijednost vertikane komponente reakcije

$$A'' = H \cdot \tan\alpha = 120 \cdot 0.66 = 79.2kN$$

pa se konačno dobija i

$$A' = A_U - A'' = 60 - 79.2 = -19.2kN$$

Na isti način proračunavamo i reakcije B' i B''

$$B'' = H \cdot \tan\alpha = 120 \cdot \frac{5.126 - 3.674}{2.2} = 79.2kN$$
$$B' = B_U - B'' = 180 - 79.2 = 100.80kN$$

Kad je poznata sila u prvom štapu možemo proračunati sve sile duž lančanice krećući se od čvora do čvora i postavljajući uslov ravnoteže $N_L \cos\alpha_L - N_D \cos\alpha_D = 0$ (odakle možemo zaključiti da je $N_L \cos\alpha_L = N_D \cos\alpha_D = H$; L-lijevo; D-desno)

Presječne sile u naznačenom presjeku

$$N = \frac{H}{\cos\left[\arctan\left(\frac{d-e}{a}\right)\right]} = \frac{120}{0.905} = 132.59kN$$

$$\sum M_2^D = 0$$
$$\Rightarrow B_U \cdot 6 \cdot 2.2 - 80 \cdot 2.2 \cdot (3+4+5) - H \cdot b + H \cdot d - U \cdot 2.2 = 0$$
$$U = -39.60kN$$

$$\sum M_1^D = 0$$
$$\Rightarrow B_U \cdot 5 \cdot 2.2 - 80 \cdot 2.2 \cdot (2+3+4)$$
$$- H \cdot (b+a) + H \cdot (e+a) + O \cdot 2.2 = 0$$
$$O = 36kN$$

$$\sum V_{II} = 0$$
$$\Rightarrow B_U - 80 \cdot 3 - H \cdot \tan\alpha_N + D \cdot \sin\beta = 0$$
$$D = 5.09kN$$

Uticajne linije

Uticajna linija za H (maksimalna vrijednost)

$$\text{“}H\text{”} = \frac{\text{“}M_G^0\text{”}}{f} = \frac{l_1 \cdot l_2}{l \cdot f} = \frac{8.8 \cdot 8.8}{17.6 \cdot 4.4} = 1.0$$

Uticajna linija za A'' (maksimalna vrijednost)

$$\text{“}A''\text{”} = H \cdot \tan\alpha = 1.0 \cdot 0.66 = 0.66$$

Uticajna linija za A'

$$\text{“}A'\text{”} = \text{“}A_U\text{”} - \text{“}H\text{”} \cdot \tan\alpha$$

Uticajna linija za N

$$\text{“}N\text{”} = \frac{\text{“}H\text{”}}{\cos\alpha_N}$$

$$_{max}\text{“}N\text{”} = \frac{1.0}{\cos\left[\arctan\left(\frac{d-e}{a}\right)\right]} = \frac{1.0}{\cos\left[\arctan\left(\frac{2.2-1.166}{2.2}\right)\right]} = 1.105$$

Uticajna linija za silu u štapu U

za silu desno od 2, ravnoteža lijevog dijela:

$$\sum M_2^L = 0 \Rightarrow A_U \cdot 2a - U \cdot a - H \cdot (b-d) = 0$$

$$U = \frac{1}{a}\left(A_U \cdot 2a - H \cdot y_2\right)$$

za silu lijevo od 2, ravnoteža desnog dijela:

$$\sum M_2^D = 0 \Rightarrow B_U \cdot 6a - U \cdot a - H \cdot (b-d) = 0$$

$$U = \frac{1}{a}\left(B_U \cdot 6a - H \cdot y_2\right)$$

Konačno, silu u štapu U možemo zapisati jedinstvenim zapisom

$$\text{“}U\text{”} = \frac{1}{a}\left(\text{“}M_2^0\text{”} - \text{“}H\text{”} \cdot y_2\right)$$

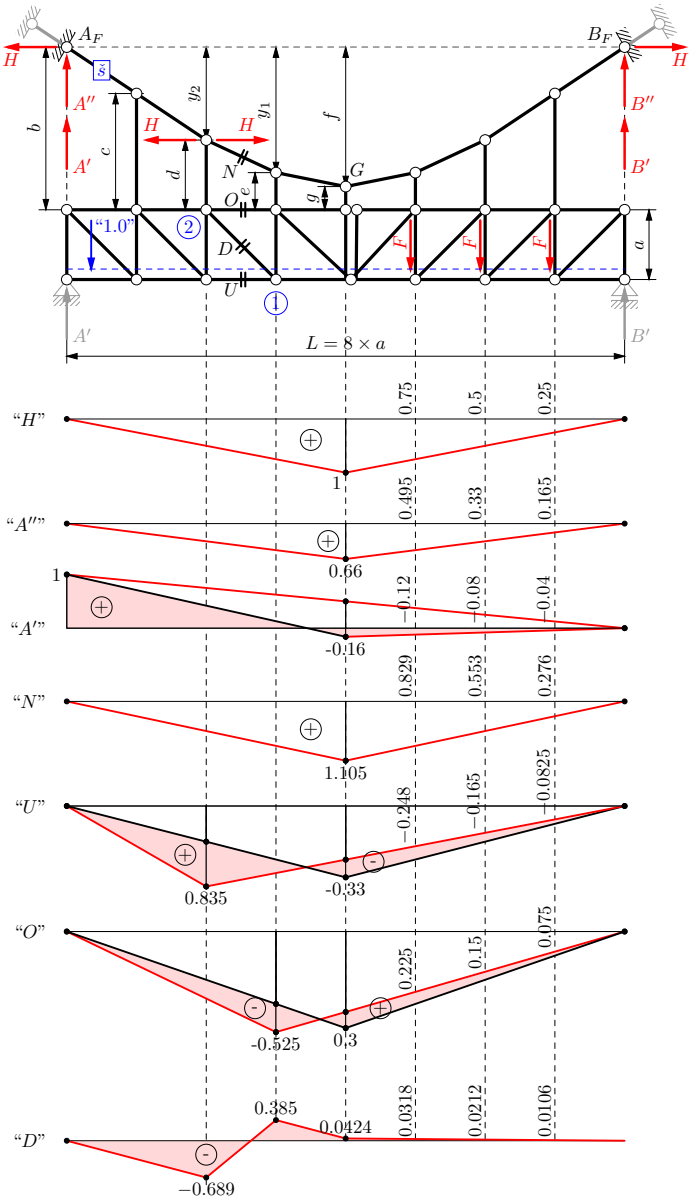
gdje je M_2^0 momenat u presjeku 2 na zamišljenoj prostoj gredi.
Za silu u štapu O

$$\text{“}O\text{”} = \frac{1}{a}\left(-\text{“}M_1^0\text{”} + \text{“}H\text{”} \cdot y_1\right)$$

Sila u štapu D

$$\text{“}D\text{”} = \frac{1}{\sin\beta}(\text{“}T_D^0\text{”} - \text{“}H\text{”} \cdot \tan\alpha)$$

gdje je “T_D^0” uticajna linija za dijagonalu kao na prostoj rešetci od A do B, β ugao nagiba dijagonale D, α ugao nagiba štapa lančanice u posmatranom presjeku.



Slika 2.59

Kontrola presječnih sila

$$\begin{aligned}
H &= 80 \cdot (0.75 + 0.5 + 0.25) &&= 120.0kN \\
A'' &= 80 \cdot (0.495 + 0.33 + 0.165) &&= 79.2kN \\
A' &= 80 \cdot (-0.12 - 0.08 - 0.04) &&= -19.2kN \\
N &= 80 \cdot (0.829 + 0.553 + 0.726) &&= 132.6kN \\
U &= 80 \cdot (-0.248 - 0.165 - 0.0825) &&= -39.6kN \\
O &= 80 \cdot (0.225 + 0.15 + 0.075) &&= 36.0kN \\
D &= 80 \cdot (0.0318 + 0.0212 + 0.0106) &&= 5.09kN
\end{aligned}$$

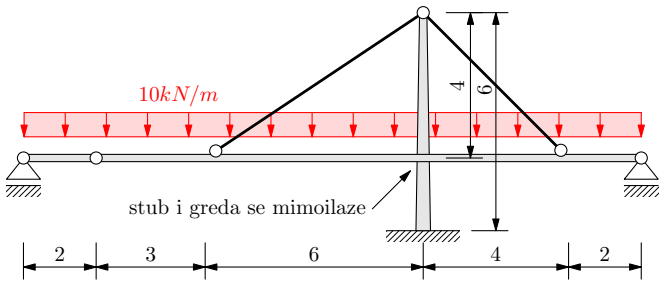
2.6 Test 8.5.2014

Zadatak 1 Za konstrukciju na slici 2.60:

a) odrediti stepen statičke neodređenosti,

b) proračunati reakcije,

c) proračunati i nacrtati dijagram momenata, poprečnih i normalnih sila.



Slika 2.60

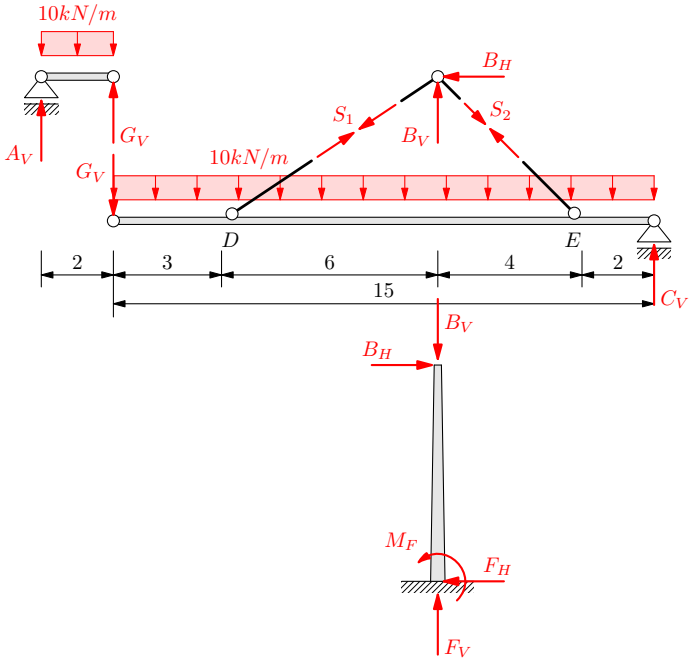
Rješenje

a) Stepen statičke neodređenosti

$$SSK = 2n - (s + c + r) = 2 \cdot 7 - (7 + 2 + 5) = 0$$

$\Rightarrow$ Statički određen nepokretan sistem.

b) Proračun reakcija



Slika 2.61

$$G_V = A_V = \frac{2 \cdot 10}{2} = 10$$

$$C_V = -\frac{G_V(3+6) + 10 \cdot 15 \cdot \left[(3+6) - \frac{15}{2}\right]}{4+2} = -52.5kN$$

$$B_H = F_H = 0$$

$$B_V = \frac{G_V \cdot 15 + 10 \cdot 15 \cdot \frac{15}{2}}{4+2} = 212.5kN$$

$$F_V = B_V = 212.5kN$$

$$M_F = 0$$

c) Presječne sile

$$S_1 \cdot \cos(\arctan\frac{4}{6}) - S_2 \cdot \cos(\arctan\frac{4}{4}) = 0$$

$$B_V - S_1 \cdot \sin(\arctan\frac{4}{6}) - S_2 \cdot \sin(\arctan\frac{4}{4}) = 0$$

$$S_2 = 1.1767 \cdot S_1$$

$$0.5547 \cdot S_1 + 0.7071 \cdot (1.1767 \cdot S_1) = B_V$$

$$S_1 = 153.24kN$$

$$S_2 = 180.31kN$$

$$M_D = -G_V \cdot 3 - 10 \cdot 3 \cdot 1.5 = -75kNm$$

$$M_E = C_V \cdot 2 - 10 \cdot 2 \cdot 1 = -125kN$$

$$T_D^L = G_V + 10 \cdot 3 = 40kN$$

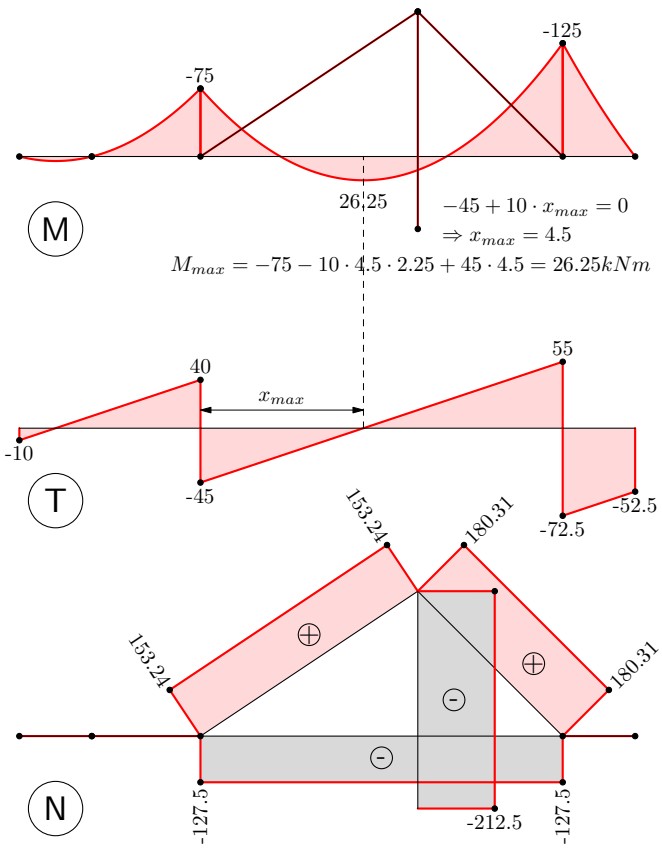
$$T_D^D = T_D^L - S_1 \cdot \sin(\arctan\frac{4}{6}) = -45kN$$

$$T_E^L = T_D^D + 10 \cdot 10 = 55kN$$

$$T_E^D = T_E^L - S_2 \cdot \sin(\arctan\frac{4}{4}) = -72.5kN$$

$$T_C = T_E^D + 10 \cdot 2 = -52.5kN$$

$$N_D = -S_1 \cdot \cos(\arctan\frac{4}{6}) = -S_2 \cdot \cos(\arctan\frac{4}{4}) = -127.5$$



Slika 2.62

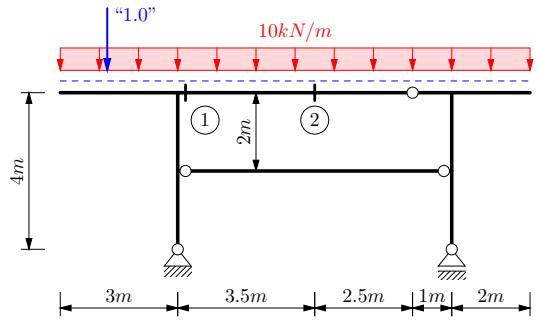
Zadatak 2 Za nosač na slici 2.63 obraditi sljedeće:

a) Provjeriti stepen statičke neodređenosti,

b) Proračunati reakcije i presječne sile u karakterističnim tačkama i nacrtati dijagrame presječnih sila,

c) Proračunati i nacrtati uticajne linije za naznačene presjeke,

d) Presječne sile dobijene pod b) kontrolisati koristeći uticajne linije dobijene pod c)



Slika 2.63

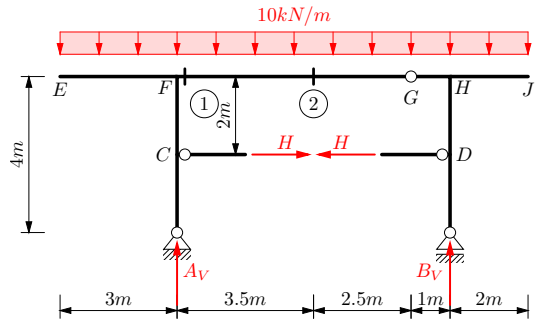
Rješenje

a) Stepen statičke neodređenosti

$SSK = 2 \cdot n - (s + c + r) = 2 \cdot 9 - (9 + 6 + 3) = 0$

$\Rightarrow$ Statički određen nepokretan nosač

b) Reakcije i presječne sile



Slika 2.64

$\overset{\curvearrowleft}{A}: \; B_V \cdot 7 - 12 \cdot 10 \cdot 3 = 0 \Rightarrow B_V = \dfrac{360}{7} = 51.43kN$

$\overset{\curvearrowright}{B}: \; A_V \cdot 7 - 12 \cdot 10 \cdot 4 = 0 \Rightarrow A_V = \dfrac{480}{7} = 68.57kN$

Provjera

$\uparrow: \; A_V + B_V - 12 \cdot 10 = 0$

Sila u zatezi H

$\overset{\curvearrowleft D}{G}: \; B_V \cdot 1 - 10 \cdot 3 \cdot 1.5 - H \cdot 2 = 0 \Rightarrow H = 3.21kN$

Presječne sile

Dio A-C

$0 \le x \le 2$

x	0	2
M	0	0
T	0	0
N	-68.57	-68.57

Dio C-F

$2 \le x \le 4$

x	2	4
M	0	-6.43
T	3.21	3.21
N	-68.57	-68.57

Dio B-D

$0 \le x \le 2$

x	0	2
M	0	0
T	0	0
N	-51.43	-51.43

Dio D-H

$2 \le x \le 4$

x	2	4
M	0	6.43
T	-3.21	-3.21
N	-51.43	-51.43

Dio E-F

$0 \le x \le 3$

x	0	3
M	0	$-10 \cdot 3 \cdot 1.5 = 45$
T	0	$10 \cdot 3 = 30$
N	0	0

Dio F-G

$3 \le x \le 9$

x	3	6.5	9
M	$-45 - 6.43$ $= -51.43$	-51.43 $+38.57 \cdot 3.5$ $-10 \cdot 3.5 \cdot 1.75$ $= 22.32$	22.32 $+3.57 \cdot 2.5$ $-10 \cdot 2.5 \cdot 1.25$ $= 0$
T	$10 \cdot 3 - 68.57$ $= -38.57$	-38.57 $+10 \cdot 3.5$ $= -3.57$	$-3.57 + 10 \cdot 2.5$ $= 21.43$
N	-3.21	-3.21	-3.21

Dio G-H

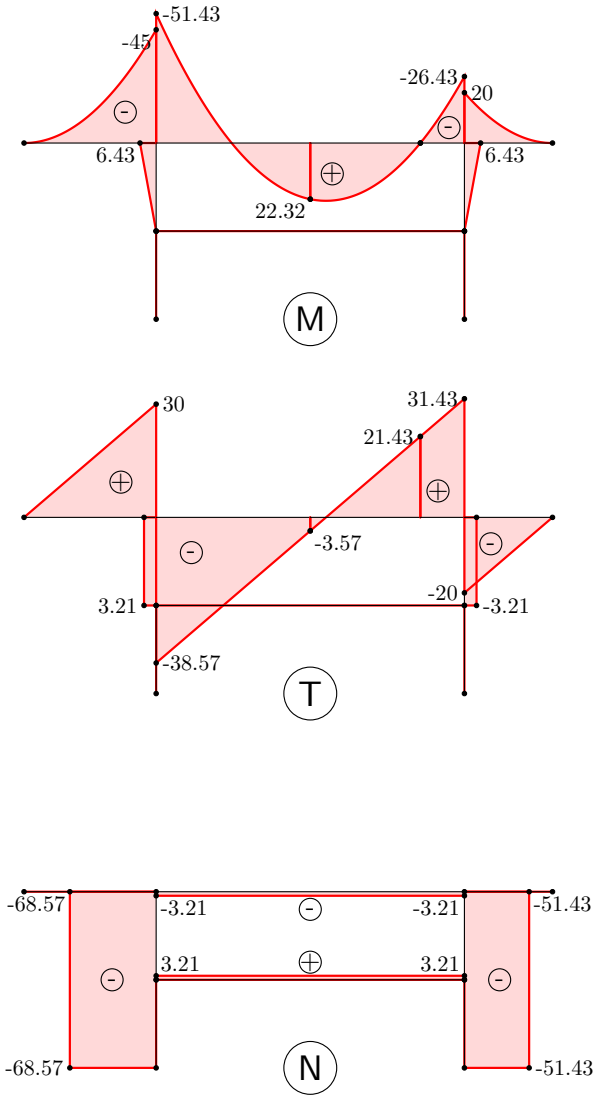
$6 \le x \le 7$

x	6	7
M	0	$-21.43 \cdot 1$ $-1 \cdot 10 \cdot 0.5$ $= -26.43$
T	21.43	$21.43 + 10$ $= 31.43$
N	-3.21	-3.21

Dio J-H

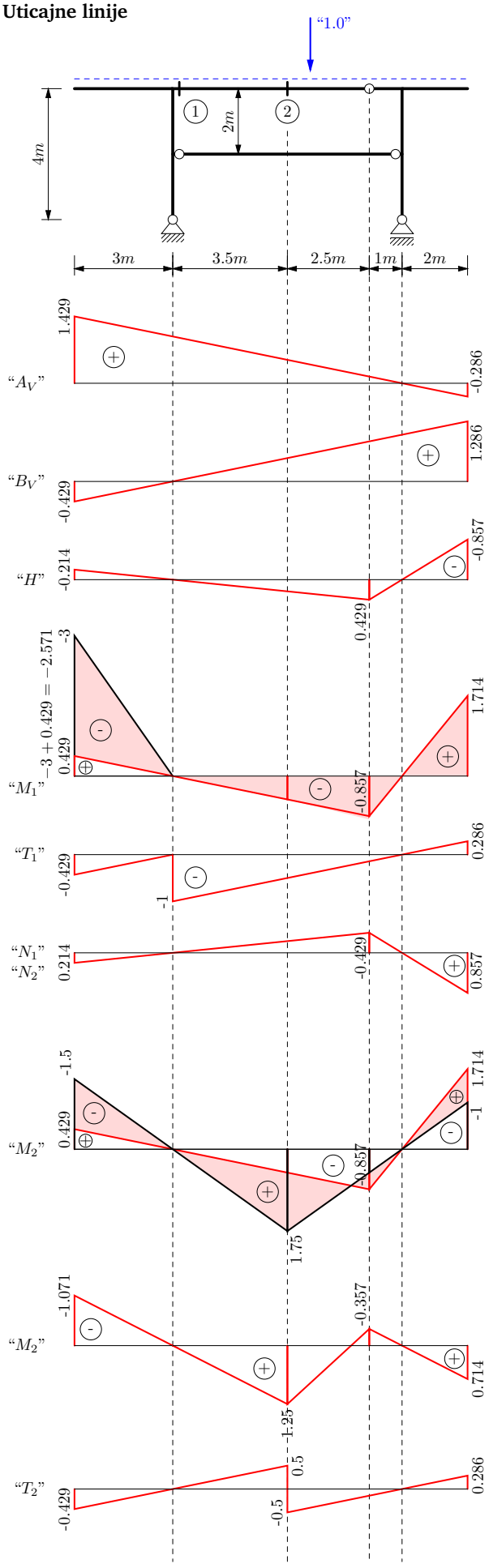
$0 \le x' \le 2$

x'	0	2
M	0	$10 \cdot 2 \cdot 1 = 20$
T	0	$10 \cdot 2 = 20$
N	0	0



Slika 2.65

c) Uticajne linije



Slika 2.66

d) Provjera presječnih sila

$$M_1 = \left(-\frac{2.571\cdot 3}{2} - \frac{0.857\cdot 6}{2} - \frac{0.857\cdot 1}{2} + \frac{1.714\cdot 2}{2}\right)\cdot 10$$
$$= -51.42kNm$$
$$T_1 = \left(\frac{-0.429\cdot 3}{2} - \frac{1\cdot 7}{2} + \frac{0.286\cdot 2}{2}\right)\cdot 10$$
$$= -38.57kN$$
$$N_1 = \left(\frac{0.214\cdot 3}{2} - \frac{0.429\cdot 6}{2} - \frac{0.429\cdot 1}{2} + \frac{0.857\cdot 2}{2}\right)\cdot 10$$
$$= -3.23kN$$

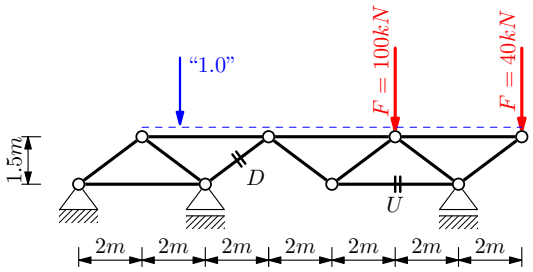
$$M_2 = \left(\frac{-1.071\cdot 3}{2} + \frac{1.25\cdot 3.5}{2} + \frac{(1.25+0.357)\cdot 2.5}{2} - 0.357\cdot 2.5 - \frac{-0.357\cdot 1}{2} + \frac{0.714\cdot 2}{2}\right)\cdot 10$$
$$= 22.32kN$$
$$T_2 = \left(\frac{-0.429\cdot 3}{2} + \frac{0.5\cdot 3.5}{2} - \frac{0.5\cdot 3.5}{2} + \frac{0.286\cdot 2}{2}\right)$$
$$= -3.57kN$$
$$N_2 = N_1$$

Zadatak 3 Za konstrukciju na slici 2.67:

a) proračunati reakcije i presječne sile u štapovima D i U,

b) proračunati i nacrtati uticajne linije u štapovima D i U,

c) kontrolisati presječne sile u štapovima D i U proračunate pod tačkom a) koristeći uticajne linije proračunate pod tačkom b).



Slika 2.67

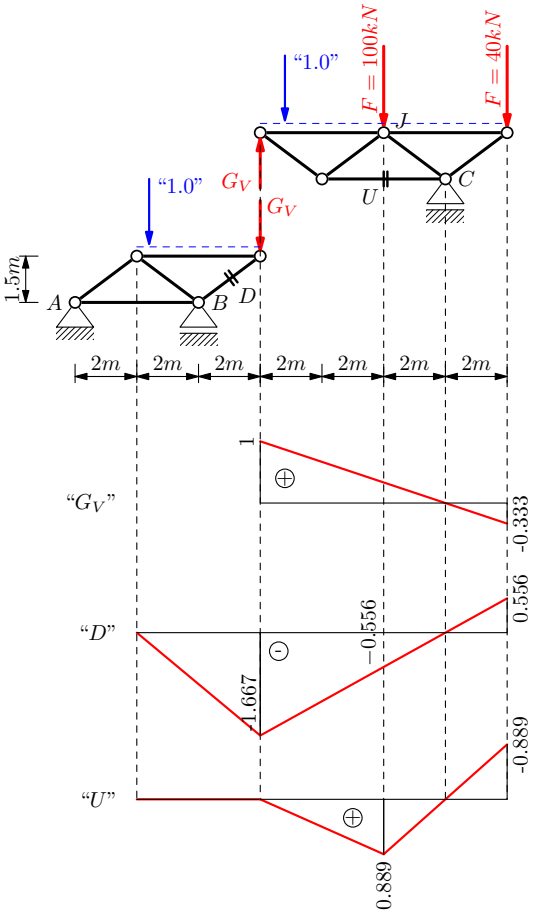
Rješenje

Reakcije

$$\overset{\curvearrowright}{C} = 0: \quad G_V\cdot 6 - 100\cdot 2 + 40\cdot 2 = 0$$
$$\Rightarrow G_V = 20kN$$
$$\overset{\curvearrowright}{G} = 0: \quad C_V\cdot 6 - 100\cdot 4 - 40\cdot 8 = 0$$
$$\Rightarrow C_V = 120kN$$
$$\overset{\curvearrowright}{A} = 0: \quad B_V\cdot 4 - G_V\cdot 6 = 0$$
$$\Rightarrow B_V = 30kN$$
$$\overset{\curvearrowright}{B} = 0: \quad A_V\cdot 4 + G_V\cdot 2 = 0$$
$$\Rightarrow A_V = -10kN$$

Presječne sile

$$D\sin\alpha + G_V = 0 \rightarrow D = -20/0.6 = -33.33kN$$
$$\overset{\curvearrowright}{J} = 0: \quad C_V\cdot 2 - 40\cdot 4 - U\cdot 1.5 = 0$$
$$U = 53.333kN$$



Slika 2.68

Proračun presječnih sila koristeći uticajne linije

$$D = 100\cdot(-0.556) + 40\cdot(0.556) = -33.33kN$$
$$U = 100\cdot 0.889 + 40\cdot(-0.889) = 53.333kN$$