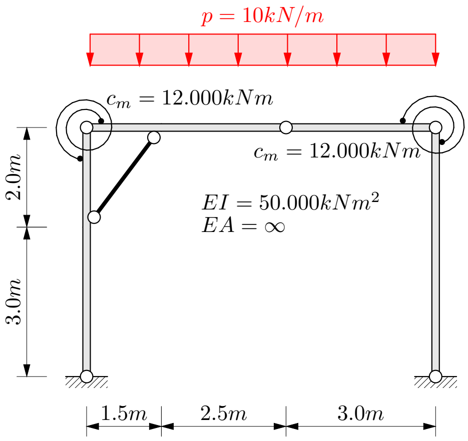
A.4 New in jlinpro - internal elastic links

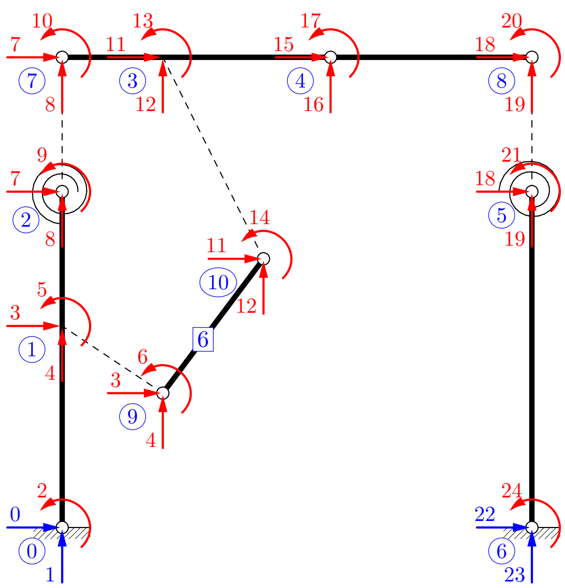
Internal links are implemented so that we can solve problems such as structure on figure A.19, that has springs in corners.



Slika A.19

In joint with spring we define two nodes, (2 and 7 left, and 5 and 8 in the right joint), see figure A.20. These two nodes in one joint, have the same translational degrees of freedom and different rotational degrees of freedom. Rotational degrees of freedom will be coupled with link elements, which have stiffness matrix

$$\boldsymbol{K} = \begin{bmatrix} k & -k \\ -k & k \end{bmatrix}$$



Slika A.20: Numbering nodes and degrees of freedom

Listing A.7: Definition of degrees of freedom

```
defdofs
0,0,1,2
1,3,4,5
2,7,8,9
3,11,12,13
4,15,16,17
5,18,19,21
6,22,23,24
7,7,8,10
8,18,19,20
9,3,4,6
10,11,12,14
enddefdofs
```

Degrees of freedom are defined with pair of `defdofs` and `enddefdofs`, and in between we put node number, dof1, dof2, dof3. For given structure dof definition is given in listing A.7

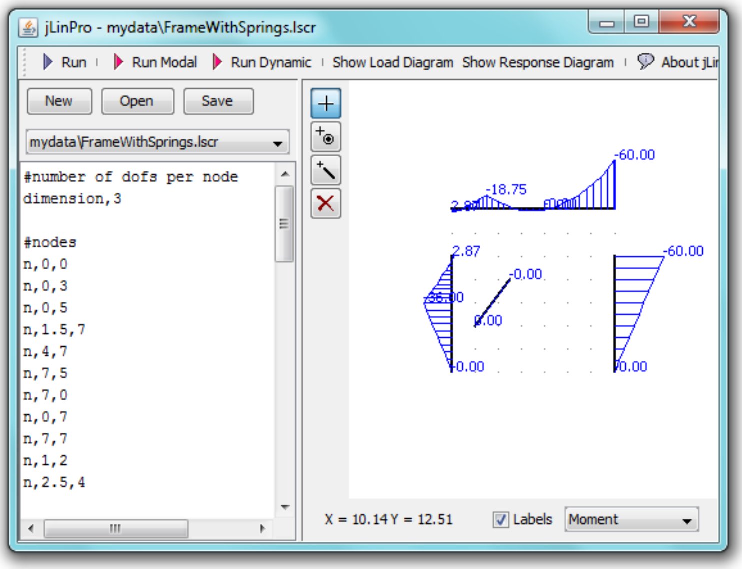
Complete input file is given in file `Framewithsprings.lscr`

Listing A.8: Complete input file, `FramewithSprings.lscr`

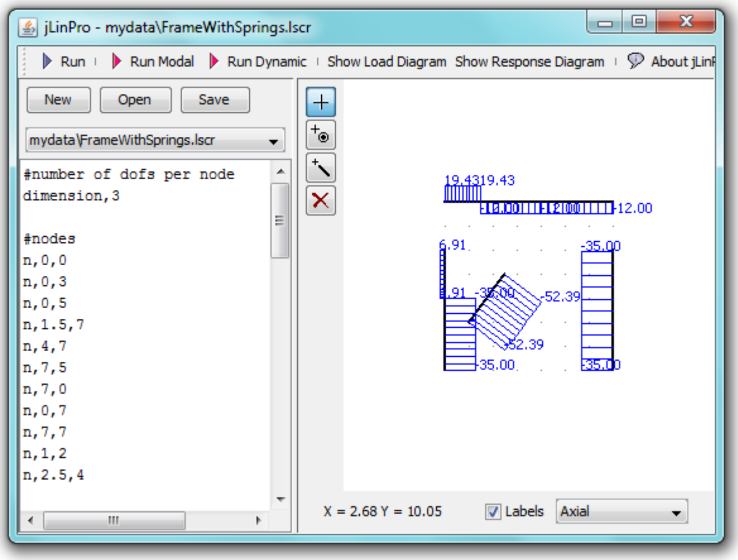
```
#number of dofs per node
dimension,3

#nodes
n,0,0
n,0,3
n,0,5
n,1.5,7
n,4,7
n,7,5
n,7,0
n,0,7
n,7,7
n,1,2
n,2.5,4

# dof definition
# in case of manuall dof definition
# dofs have to be defined for each node

defdofs
0,0,1,2
1,3,4,5
2,7,8,9
3,11,12,13
4,15,16,17
5,18,19,21
6,22,23,24
7,7,8,10
8,18,19,20
9,3,4,6
10,11,12,14
enddefdofs

#cross sections
cs,50000,1E6,1

#elements
beam2D,0,1
beam2D,1,2
beam2D,7,3
beam2D,3,4,release,mj
beam2D,4,8
beam2D,5,6
beam2D,9,10

#links
#link,nodei,nodej,localdof,stiffness
link,2,7,2,12000
link,5,8,2,12000

#supports
#spring,node,localdof,stiffness
spring,0,0,1E10
spring,0,1,1E10
spring,6,0,1E10
spring,6,1,1E10

#loads
py,-10

#load application
al,0,2,3,4
```

Run calculation

Now since for calculation of beam element stiffness matrix only length is calculated from position of nodes, and lengths are correct compared to original structure, **and since** link stiffness matrix is *not* calculated using any length but stiffness is inserted directly, **and since** global structural stiffness matrix is assembled based on DOF numbers that we defined manually, we expect jlinpro to be able to solve such *split* structure correctly, so push button **Run**.

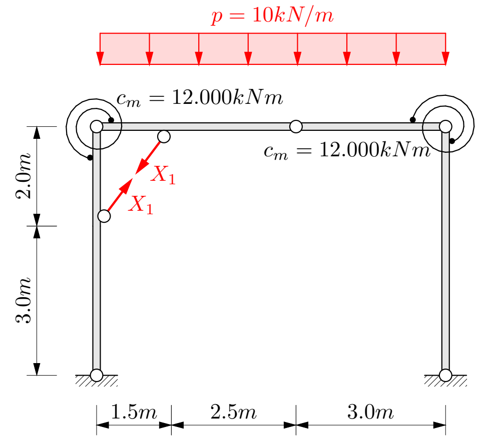


Slika A.21: Moment diagram



Slika A.22: Axial force diagram

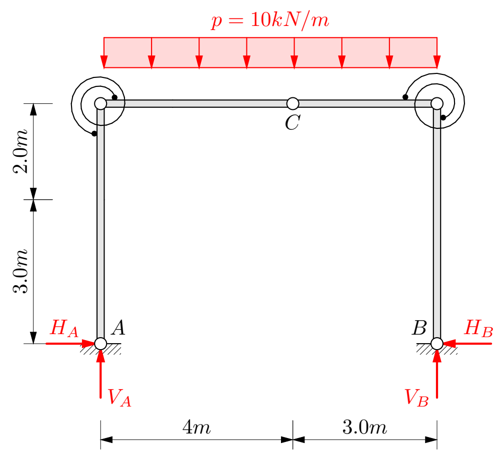
Now we will calculate this structure by the Force method in order to compare results.



Slika A.23: Primary system

Rješenje Primary system is shown on figure A.23. Reactions

$$\overset{\frown}{A}: V_B \cdot 7 - 10 \cdot 7 \cdot 3.5 = 0$$
$$\Rightarrow V_B = 35kN$$
$$\overset{\frown}{B}: V_A \cdot 7 - 10 \cdot 7 \cdot 3.5 = 0$$
$$\Rightarrow V_A = 35kN$$
$$\overset{\frown}{C_D}: V_B \cdot 3 - 10 \cdot 3 \cdot 1.5 - H_B \cdot 5 = 0$$
$$\Rightarrow H_B = 12kN$$
$$\overset{\frown}{C_L}: V_A \cdot 4 - 10 \cdot 4 \cdot 2 - H_A \cdot 5 = 0$$
$$\Rightarrow H_A = 12kN$$



Slika A.24

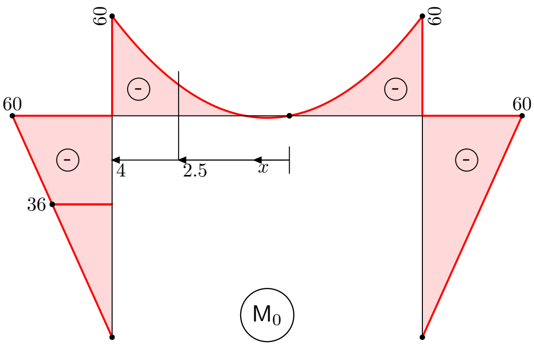
Moment diagram from external load is shown on figure A.25 and from unit force on figure A.26

Calculation of flexibility coefficients

In order to calculate Δ_{10} we have to calculate integral

$$\int_l \frac{M_0 \overline{M}_1}{EI}$$

that in range from 2.5 to 4 is not given in tables, (see figure A.25) so we will calculate it without tables. Coordinate system is placed in hinge with x-axis pointing left, because it is easier to write moment function. Shear force in hinge is $5kN$.



Slika A.25: Bending moment diagram from external loading.

Bending moment function is

$M_0(x) = 5 \cdot x - 5 \cdot x^2$

We calculate area and centroid position.

$$A_{M_0} = \left| \int_{2.5}^{4} (5 \cdot x - 5 \cdot x^2)\,\mathrm{d}x \right| = 56.25$$

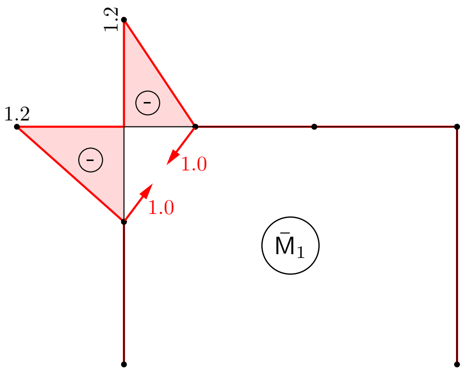
$$x_T^A = \frac{\int_{2.5}^{4} x \cdot (5 \cdot x - 5 \cdot x^2)\,\mathrm{d}x}{A} = \frac{190.547}{56.25} = 3.3875$$

finally

$$\frac{1}{EI} \int_{2.5}^{4} M_0 \overline{M}_1 = \frac{1}{EI} A_{M_0} \cdot \overline{M}_1(x_T^A) =$$
$$\frac{1}{EI} 56.25 \cdot \frac{1.2}{1.5} \cdot (3.3875 - 2.5) = \frac{1}{EI} 39.938$$

The rest of calculation of flexibility coefficient can be calculated using tables

$$EI\Delta_{10} = \frac{2 \cdot 1.2}{6}(36 + 2 \cdot 60) + EI \cdot \frac{60 \cdot 1.2}{c_M} + \underbrace{39.938}_{\text{već proorač.}}$$
$$= 62.4 + 300 + 39.938 = 402.338$$



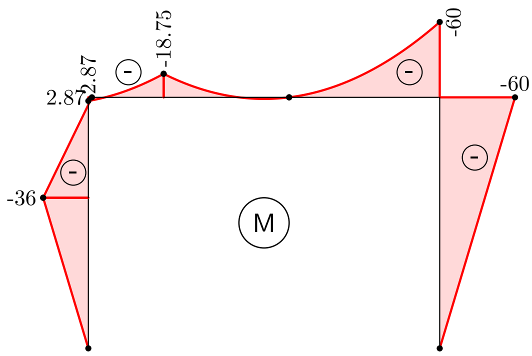
Slika A.26: Bending moment from unit force

Coefficient δ_{11} is calculated using tables

$$EI\delta_{11} = \frac{1}{3} \cdot 2 \cdot 1.2 \cdot 1.2 + \frac{1}{3} \cdot 1.5 \cdot 1.2 \cdot 1.2 + EI \frac{\overline{M}_1^C \overline{M}_1^C}{c_m} = 7.68$$

Finally we obtain unknown force

$$X_1 = -\frac{402.338}{7.68} = -52.388kN$$



Slika A.27: Bending moment diagram